PACIFIC JOURNAL OF MATHEMATICS
Vol. 119, No. 1, 1985

ON BASES IN STRICT INDUCTIVE AND PROJECTIVE LIMITS OF LOCALLY CONVEX SPACES

KLAUS FLORET AND VINCENZO BRUNO MOSCATELLI

This note investigates, for certain locally convex spaces which have bases and are strict inductive or projective limits, the structural property of being a direct sum or a product. Our approach is based on a suitably more general version of a decomposition lemma originally due to S. Dineen and gives a better understanding of the non-existence of bases in certain nuclear (F)- and strict (LF)-spaces. Our method also allows us to investigate the structure of various other non-nuclear spaces with unconditional bases yielding, in particular, examples of spaces with no such bases. In part, this also motivated us to include some rather general remarks on the problem of when the strong dual of a homomorphism between locally convex spaces is a homomorphism as well.

There are nuclear spaces of type (F) and proper strict (LF) which do not admit bases. The first example of such a Fréchet space is due to Mitiagin and Zobin [**18**] in 1974. Later, in 1979, Dubinsky [**6**] constructed a Fréchet space (with a continuous norm) which does not even have the bounded approximation property; a very simple construction of spaces of this type was recently given by Vogt [**23**]. A totally different approach in constructing nuclear Fréchet spaces without bases was presented by the second author [**19**] in 1980. A nuclear, proper, strict (LF)-space without a basis is exhibited in the paper [**8**] of the first author: this space has even the property that all stepspaces are nuclear Fréchet spaces with continuous norms and without the bounded approximation property. All the proofs have in common the fact that they show the non-existence of a basis by checking that the space under consideration does not have a property it should have if it had a basis, such as being countably normed [**6**], [**23**], being a product space [**19**], or having a continuous norm [**8**].

It is one of the purposes of this note[1] to contribute to a better understanding of certain nuclear spaces of type (F) and (LF), with respect to admitting or not a basis, by using the structural property of being a product or a direct sum. At the same time, this approach will free the construction in [**19**] from a somehow not quite natural use of Dubinsky's classification of perfect Fréchet sequence spaces [**5**]. The key to our

[1] Part of the results were already presented at the 17° Seminário de Análise [**9**].