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ON THE MINORANT PROPERTIES IN $C_p(H)$

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We improve in two directions a recent result of B. Simon about the minorant property in $C_p(H)$; the methods also allow us to extend a result of H. Shapiro and to obtain an apparently new result on matrices with positive entries.

Introduction. Let H be a complex Hilbert space, which will always be the space l^2 of square summable sequences or the space l_n^2 of all n-tuples of complex numbers with the hermitian norm, equipped once and for all with an orthogonal basis $(e_i)_{i\in I}$ (I finite or countable). Let $K(H)$ be the set of all compact operators of H; if $C \in K(H)$, put $|C| = \sqrt{C^*C}$ and let $\mu_1(C), \mu_2(C),\dots,\mu_i(C)$ be the eigenvalues of $|C|$, rearranged in decreasing order; if $1 \le p < \infty$, put

$$\|C\|_p = \Big(\sum_{i\in I}(\mu_i(C))^p\Big)^{1/p} = \big(\mathrm{Tr}|C|^p\big)^{1/p} = \big[\mathrm{Tr}(C^*C)^{p/2}\big]^{1/p}$$

(where for $A \in K(H)$, $\mathrm{Tr}\,A \overset{\text{def}}{=} \sum_{i\in I}\langle Ae_i, e_i\rangle$ is the trace of A whenever it exists).

Let $C_p(H)$ be the set of all $C \in K(H)$ such that $\|C\|_p < \infty$, ($C_\infty(H) = K(H)$ and $\|C\|_\infty = \mu_1(C)$ is the usual operator norm of C). It is well known that $C_p(H)$, with the norm $\|\ \|_p$, is a Banach space ([**11**]).

For $C \in K(H)$, we put

$$c_{ij} = \langle C(e_j), e_i\rangle = \mathrm{Tr}\big(C\cdot(e_i\otimes e_j)\big) = \hat{C}(i, j).$$

In the last inequality, c_{ij} is considered as a Fourier coefficient with respect to the orthonormal (in the Hilbert–Schmidt sense) system $(e_i \otimes e_j)_{(i,j)\in I\times J}$ and this allows us to keep the analogy with the commutative case ([**3**], [**4**]) in the definitions below (recall that $e_i \otimes e_j$ is the operator of rank one defined by:

$$(e_j \otimes e_j)(x) = \langle x, e_i\rangle e_j).$$

DEFINITION 1. If $A, B \in K(H)$, we say that A is a minorant of B if $|a_{ij}| \le b_{ij}$ for $(i, j) \in I \times J$, that is if $|\hat{A}| \le \hat{B}$. We say that $C_p(H)$ has the minorant property, and we abbreviate this to (m)-property, if

$$A, B \in C_p(H) \quad \text{and} \quad |\hat{A}| \le \hat{B} \Rightarrow \|A\|_p \le \|B\|_p.$$