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NON-ISOTROPIC UNITARY SPACES AND MODULES WITH CAUCHY-SCHWARZ INEQUALITIES

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This subject is concerned with non-isotropic unitary spaces V over involutorial division rings D with characteristic not 2 and with non-trivial non-archimedean exponential valuations w, which are abelian. It will require a generalized Cauchy-Schwarz inequality relative to w. The dimension of V over D need not be finite. Treatments of the unitary module V_0 of finite vectors v in v (finite, in a technical sense), the ring L_0 of linear transformations of V that increase lengths, and the unitary group U yield information on the normal subgroup structure of this group and the factor group $U^{(r)}/U^{(r)} \cap Z$, where $U^{(r)}$ is the rth derived group of U and Z is the center of the ground division ring D.

Introduction. From a purely ring-theoretic viewpoint this subject arose from the treatment of primitive ring with involution L, in which, 2 is invertible and $1 - k$ is invertible for every skew-symmetric k in L. These invertibility assumptions ensure plenty of unitary elements $u = u^{*-1}$ in L, via the Cayley transform $u^{(k)} = (1 - k)/(1 + k)$ and one is interested in deciding whether or not the factor group $[U, U]/[U, U] \cap Z$ is simple, where $Z = \text{center}(L)$ and U is the group of unitary elements in L. Another question which is of interest to me is the nature of the ring that is generated by U. From a more down to earth viewpoint, this subject specializes to the rings L of the form $L = L(V)$, the full ring of linear transformations of a certain left vector space V. It will be assumed throughout that V is a non-isotropic unitary space (in the sense of I. Kaplansky), where the involutorial ground division ring $(D; *)$ will be equipped with a non-trivial non-archimedean exponential valuation w, which is abelian. In fact, w will be a *$*$-valuation* (in the sense of S. S. Holland, Jr.). I will require, furthermore,

$$(1) \qquad 2w(u \cdot v) \geq w(u \cdot u) + w(v \cdot v) + 2\varepsilon_0,$$

where $(\cdot)$ is the form of the unitary space $V, \cdots \leq \cdots$ is the ordering in G, the value group of w, and ε_0 is a constant (depending on V) in G. As shown by a theorem of Kaplansky, if L is as in the outset then L can be represented as a subring of $L(V)$, where the involution in L corresponds to the adjoint involution $\phi \to \phi^*$, provided L has a minimal left ideal. Of