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IWASAWA THEORY FOR THE ANTICYCLOTOMIC EXTENSION

RODNEY I. YAGER

We compute the structure of local units modulo elliptic units for the anticyclotomic $\mathbf{Z}_p$-extension of an imaginary quadratic field with class number one.

Introduction. Let K be an imaginary quadratic field with discriminant $-d_K$ and, for simplicity, class number one. We let p be a rational prime which splits in K, and write K_∞^- for the anticyclotomic $\mathbf{Z}_p$-extension of K, the unique $\mathbf{Z}_p$-extension of K unramified outside p such that the action of complex conjugation c on $\Gamma^- = \mathrm{Gal}(K_\infty^-/K)$ is given by

$$c \cdot \tau = c\tau c^{-1} = \tau^{-1}.$$

Let K_n^- denote the n-th layer of the extension K_∞^- over K. It is clear that both primes of K dividing (p) share the same inertia group for the extension K_n^- over K, which is unramified outside p. Under our assumption that K has class number one, it follows that both primes are totally ramified in K_n^-. Choose one of the primes $\mathfrak{p}$ of K dividing (p), and denote by U_n the group of principal units (i.e. those congruent to one modulo the maximal ideal) of the completion of K_n^- at the unique prime above $\mathfrak{p}$. The natural embedding of K_n^- in its completion sends the group of principal global units E_n of K_n^- into U_n, and we write $\bar{E}_n$ for the $\mathbf{Z}_p$-submodule of U_n which they generate. The $\mathbf{Z}_p[[\Gamma^-]]$-module $X_\infty = \varprojlim U_n/\bar{E}_n$, where the projections are the norm maps, clearly is important in the arithmetic of K, as it is the Galois group of the maximal abelian p-extension of K_∞^- unramified outside $\mathfrak{p}$, or equivalently, the $\mathfrak{p}$-primary part of the idèle class group of K_∞^-.

The $\mathbf{Z}_p[[\Gamma^-]]$-module X_∞ becomes a torsion $\lambda = \mathbf{Z}_p[[T]]$-module in the usual way if we fix a topological generator τ of Γ^- and define the action of T by setting

$$T \cdot x = (\tau - 1) \cdot x.$$

The classification theorem for torsion λ-modules shows that there is a unique set of principal λ-ideals $\{\mathscr{F}_1, \ldots, \mathscr{F}_r\}$ such that there is a λ-homomorphism $X_\infty \to \oplus_{i=1}^r \lambda/\mathscr{F}_i$ with finite kernel and co-kernel. Moreover,