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THE BIGGER BRAUER GROUP AND ÉTALE COHOMOLOGY

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The classical Brauer group $B(R)$ is formed from equivalence classes of Azumaya algebras over the ring R. The bigger Brauer group $\tilde{B}(R)$ is formed in a similar way from equivalence classes in a larger category of R-algebras. This larger category is defined through axioms similar to those defining Azumaya algebras but with the requirement for an identity dropped. In this paper we identify $\tilde{B}(R)$ with the second étale cohomology of Spec(R) (with G_m as coefficients). The classical Brauer group consists of the torsion subgroup of this cohomology group. This result yields a concrete realization of second étale cohomology and also enables us to settle several questions about the relation of $\tilde{B}(R)$ to $H^2(\Delta, \mathbf{Z})$ in the case where R is a Banach algebra with maximal ideal space Δ.

That $B(R)$ may indeed be a proper subgroup of $\tilde{B}(R)$ is demonstrated by the fact that there is an isomorphism $\tilde{B}(R) \to H^3(\Delta, \mathbf{Z})$ if $R = C(\Delta)$ for a compact Hausdorff space Δ (cf. Prop. 6.6 of **[12]**). Since $B(R)$ is carried to the torsion subgroup of $H^3(\Delta, \mathbf{Z})$ by this map, $B(R)$ and $\tilde{B}(R)$ will be distinct if $H^3(\Delta, \mathbf{Z})$ is non-torsion. In the case $R = C(\Delta)$ the central separable algebras are related to another class of R-algebras — the algebras with continuous trace from C^*-algebra theory. In fact, in **[11]** we use an elementary proof of the surjectivity of the map $\tilde{B}(C(\Delta)) \to H^3(\Delta, \mathbf{Z})$ to give an elementary proof of the existence of continuous trace C^*-algebras of given Dixmier-Douady class (cf. **[5]**).

The map $\tilde{B}(R) \to H^3(\Delta, \mathbf{Z})$ is defined for any Banach algebra R with maximal ideal space Δ, but in general neither the injectivity nor surjectivity of this map was established in **[12]**. It was proved to be an injection on the sub-group $\overline{B}(R) \subset \tilde{B}(R)$ consisting of equivalence classes containing an algebra finitely presented as an R-module. The functor $\overline{B}$ agrees with $\tilde{B}$ on Noetherian rings and is continuous (commutes with direct limit) whereas $\tilde{B}$ was not proved to be continuous in **[12]**. In general, $B(R) \subset \overline{B}(R) \subset \tilde{B}(R)$. However, the following questions were left unanswered in **[12]**: Is $\overline{B}(R)$ always equal to $\tilde{B}(R)$? Is $\overline{B}(R)$ always $B(R)$? (This would force $B(R) = \tilde{B}(R)$ in the Noetherian case — a possibility left open in **[12]**.) When R is a commutative Banach algebra with maximal ideal space Δ, is $\tilde{B}(R) \to H^3(\Delta, \mathbf{Z})$ always surjective? Is it always injective? Is $\overline{B}(R) \to H^3(\Delta, \mathbf{Z})$ always surjective?