PACIFIC JOURNAL OF MATHEMATICS
Vol. 119, No. 2, 1985

A MAXIMAL FUNCTION CHARACTERIZATION OF A CLASS OF HARDY SPACES

ROBYN OWENS

In this paper we obtain a maximal function characterisation of a class of Hardy spaces H^p which are defined on the upper half plane and combine many of the properties of the classical Hardy spaces for the half plane and the unit disc.

Burkholkder, Gundy and Silverstein [1] have shown that the classical Hardy spaces $H^p(D)$ and $H^p(P)$ on the unit disc D and the upper half plane P can be characterized by the maximal function. For the unit disc they proved the following: if u is an harmonic function in D and $\Omega_\alpha(\theta)$ is the Stoltz domain given by the interior of the smallest convex set containing the disc $\{z\colon |z| < \alpha\}$ and the point $e^{i\theta}$, then $u = \operatorname{Re} F$ for some $F \in H^p(D)$ if and only if $\sup\{|u(z)|\colon z \in \Omega_\alpha(\theta)\} \in L^p(\partial D)$, where ∂D denotes the boundary of D. This theorem extends an earlier one due to Hardy and Littlewood [9, I, p. 278].

The importance of this result is that a real-valued classification of these spaces of analytic functions allows the notion of Hardy space to be extended to real Euclidean spaces of arbitrary dimension. This was precisely the task undertaken by Fefferman and Stein [2] and in so doing they presented many other equivalent real-valued function characterizations of the corresponding H^p spaces.

In the 1950's [5] Hardy spaces were generalized in other directions and different measures on the boundary $\mathbf{R} = \partial P$ were considered. In this paper we will consider a class of Hardy spaces H^p which combines many of the properties of the classical Hardy spaces of the upper half plane and the unit disc. Our H^p spaces will consist of functions that are analytic in the upper half plane and yet constrained by a bounded measure on its boundary $\mathbf{R}$. This measure arises naturally [5] in the theory of abstract harmonic analysis, where the unit circle is replaced by any abelian locally compact group G and the set of indices over which one forms a "trigonometric series" is taken to be the dual group of G. In our case, the group $\mathbf{R}$ of real numbers endowed with the discrete topology is considered; its dual group can then be identified as $b\mathbf{R}$, the Bohr compactification of $\mathbf{R}$ [3].