PACIFIC JOURNAL OF MATHEMATICS
Vol. 119, No. 2, 1985

ZERO SETS OF INTERPOLATING BLASCHKE PRODUCTS

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For a function h in H^∞, $Z(h)$ denotes the zero set of h in the maximal ideal space of $H^\infty + C$. It is well known that if q is an interpolating Blaschke product then $Z(q)$ is an interpolation set for H^∞. The purpose of this paper is to study the converse of the above result. Our theorem is: If a function h is in H^∞ and $Z(h)$ is an interpolation set for H^∞, then there is an interpolating Blaschke product q such that $Z(q) = Z(h)$. As applications, we will study that for a given interpolating Blaschke product q, which closed subsets of $Z(q)$ are zero sets for some functions in H^∞. We will also give a characterization of a pair of interpolating Blaschke products q_1 and q_2 such that $Z(q_1) \cup Z(q_2)$ is an interpolation set for H^∞.

Let H^∞ be the space of bounded analytic functions on the open unit disk D in the complex number plane. Identifying a function h in H^∞ with its boundary function, H^∞ becomes the (essentially) uniformly closed subalgebra of L^∞, the space of bounded measurable functions on the unit circle ∂D. A uniformly closed subalgebra B between H^∞ and L^∞ is called a Douglas algebra. We denote by $M(B)$ the maximal ideal space of B. Identifying a function h in B with its Gelfand transform, we regard h as a continuous function on $M(B)$. Sarason **[10]** proved that $H^\infty + C$ is a Douglas algebra, where C is the space of continuous functions on ∂D, and $M(H^\infty) = M(H^\infty + C) \cup D$. For a function h in H^∞, we denote by $Z(h)$ the zero set in $M(H^\infty + C)$ for h, that is,

$$Z(h) = \{x \in M(H^\infty + C); h(x) = 0\}.$$

For a subset E of $M(H^\infty)$, we denote by $\mathrm{cl}(E)$ the weak*-closure of E in $M(H^\infty)$. A closed subset E of $M(H^\infty)$ is called an interpolation set for H^∞ if the restriction of H^∞ on E, $H^\infty|_E$, coincides with $C(E)$, the space of continuous functions on E. For points x and y in $M(H^\infty)$, we put

$$\rho(x, y) = \sup\{|f(x)|; f \in H^\infty, \|f\| \leq 1, f(y) = 0\}.$$

We note that if z and w are points in D, $\rho(z, w) = |z - w|/|1 - \overline{w}z|$, which is called the pseudo-hyperbolic distance on D. For a point x in $M(H^\infty)$, we put

$$P(x) = \{y \in M(H^\infty); \rho(x, y) < 1\},$$