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ON THE LOCATION OF ZEROES OF OSCILLATORY SOLUTIONS OF $y^{(n)} = c(x)y$

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We locate the zeroes of oscillatory solutions of wide classes of differential equations, $y^{(n)} = c(x)y$. Asymptotic techniques are used. The asymptotic behaviour of solutions and their derivatives up to the nth order are also provided.

New results are obtained in addition to old results becoming more transparent.

1. Introduction. The main purpose of this paper is to demonstrate a method for *locating* the zeroes of oscillatory solutions of the differential equation

$$y^{(n)} = c(x)y. \tag{1.1}$$

As shown by the references cited, the differential equation (1.1) attracted a considerable amount of attention. However, the location of zeroes of oscillatory solutions of (1.1) does not seem to be available in the literature. It is the purpose of this paper to fill this gap for a wide class of differential equations (1.1).

The method to be used exploits concepts of classical asymptotics which seem to us the most appropriate ones to handle problems of singular differential equations. The singularity of the differential equation (1.1) stems from the fact that the independent variable x ranges on an infinite interval and also from the fact that $c(x)$ may be unbounded.

We do assume an amount of smoothness on the coefficient $c(x)$ which is more restrictive than a continuity assumption made e.g. by Kim **[12]**. However, this is a reasonable price to be paid for obtaining the fine structure of $y^{(\nu)}(x)$, $\nu = 0, 1, \ldots, n-1$ as $x \to \infty$.

In particular, most of the asymptotic properties known so far on oscillatory and nonoscillatory solutions of (1.1) can be better understood by the techniques employed in this work.

An additional reward of this paper is that we produce Prufer type representatives for solutions of (1.1) which belong to certain subspaces of the linear space of solutions of (1.1). See e.g. Hille **[9]** p. 394.

The course of events in this paper will be as follows: After this introduction we proceed to §2, which contains preparations for an