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GELFAND THEORY IN ALGEBRAS OF DIFFERENTIABLE FUNCTIONS ON BANACH SPACES

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Dedicated to J. M. Ortega

We study algebras of differentiable functions on a reflexive Banach space, defined by polynomial approximation on bounded sets. We find the spectra of such algebras and we investigate the structure of their closed ideals. In relation to this, we treat also an approximation problem of functions f such that $f, f^1, \dots, f^{(m)}$ vanish on a weakly compact set by a method involving radical algebras and the Ahlfors-Heins theorem.

Introduction. We denote by E a complex reflexive Banach space with closed unit ball Ω, by E' its topological dual, and by m a fixed non-negative integer. If S is a subset of E and Y a Banach space, a function $g\colon S \to Y$ is said to be weakly continuous if it is continuous with respect to the weak topology on S and the norm topology on Y. We denote by $C_{wb}^m(\Omega)$ the algebra of the m times continuously differentiable complex valued functions whose m derivatives can be extended by continuity to the boundary of Ω and, moreover, these derivatives are weakly continuous on Ω. Let τ^m be the topology on $C_{wb}^m(\Omega)$ of m-uniform convergence on Ω. Equipped with τ^m, $C_{wb}^m(\Omega)$ becomes a Banach algebra.

We are interested in functions obtained by polynomial approximation. For this, we consider $A^m(\Omega)$, defined as the τ^m-completion of P_f in $C_{wb}^m(\Omega)$. Here P_f denotes the algebra of the polynomials of finite type on E, i.e. the ones generated by the elements of E' and their conjugates, and the constant functions; it is clear that $P_f \subset C_{wb}^m(\Omega)$. We may define also the spaces $A^\infty(\Omega)$, $A^m(E)$, $A^\infty(E)$ in a similar manner to $A^m(\Omega)$ (definitions below). In fact, the functions of type of $A^m(\Omega)$ have been introduced in the literature in relation with approximation questions ([**1**], [**2**], [**16**]). For instance, when E' has the bounded approximation property, $A^m(\Omega) = C_{wb}^m(\Omega)$ for every m (see [**2**]). Also, the weak continuity has been treated recently in great detail (see [**3**]), for instance). Here we study the spaces $A^m(\Omega)$, $A^\infty(\Omega)$, $A^m(E)$, $A^\infty(E)$ as topological algebras.

In §1 the Gelfand's theory in the Banach algebra $A^m(\Omega)$ is given: we show that Ω is the spectrum of $A^m(\Omega)$ and the identification between the