PACIFIC JOURNAL OF MATHEMATICS
Vol. 120, No. 1, 1985

EXTENSION OF THE HARDY-LITTLEWOOD-FEFFERMAN-STEIN INEQUALITY

AKIHITO UCHIYAMA

We will show inequalities concerning the functions of the form $f * t^{-n}\varphi(\cdot/t)(x)$ defined on R_+^{n+1} and give their applications to real Hardy spaces. These inequalities can be regarded as weak extensions of the Hardy-Littlewood-Fefferman-Stein inequality concerning harmonic functions.

1. Introduction. In C. Fefferman and E. M. Stein [6] (p. 172 Lemma 2), (see also Hardy and Littlewood [8]), they showed

THEOREM 1.A. *Let $u(x)$ be a complex-valued harmonic function defined on*

$$B = \left\{ x = (x_1, \ldots, x_n) \in R^n \colon \sum_{j=1}^{n} x_j^2 < 1 \right\}.$$

Let $p > 0$. Then

$$|u(0)|^p \le C \int_B |u(x)|^p \, dx,$$

where C is a constant depending only on p and n.

Consequently, if $u(x,t)$ is harmonic on $R_+^{n+1} = \{(x,t)\colon x \in R^n, t > 0\}$ and if $p > 0$, then we have

$$(1.1) \qquad |u(0,1)|^p \le C \int_{|x|<1} dx \int_{1/2}^{3/2} |u(x,t)|^p \, dt.$$

This inequality has some interesting applications to the theory of real Hardy spaces. (See [6].)

In this paper we show analogous inequalities for functions of the form $f * t^{-n}\varphi(\cdot/t)(x)$ defined on R_+^{n+1}, where $f \in \bigcup_{1 \le p \le +\infty} L^p(R^n)$ is arbitrary and where $\varphi \in C(R^n) \cap \bigcap_{1 \le p \le +\infty} L^p(R^n)$ satisfies certain conditions. Our results have weaker forms than (1.1) but still they have some interesting applications to real Hardy spaces.

First we prepare several definitions.