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THE NUMBER OF EQUATIONS DEFINING POINTS IN GENERAL POSITION

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Bounds are established for the number of generators of the graded homogeneous ideal of a set of points in generic or in uniform position in the projective plane. For $n \leq 11$, n points in uniform position must have the "general" number of generators. It is shown by example that this fails for $n = 12$.

Introduction. Let Z be a set of points in $\mathbf{P}_k^2$, k algebraically closed. We say the points of Z lie in generic position if Z imposes independent conditions on curves containing it. If this holds for all subsets of Z we say that Z lies in uniform position. Given a set Z in one of these types of "general position", one would like to count the number of equations needed to cut out Z, or more precisely, the minimal number of generators ν of the graded homogeneous ideal $I(Z)$.

This question has arisen most recently in calculations of the Cohen-Macaulay-type of singularities. For example, it is shown in [7] that if A is the local ring at a curve singularity P in $\mathbf{A}_k^3$, and if the lines of the tangent cone at P correspond to a set of distinct points Z in generic position in $\mathbf{P}_k^2$, then the Cohen-Macaulay-type of A is equal to $\nu(I(Z)) - 1$. It is then natural to look for geometric conditions on Z which will allow the Cohen-Macaulay-type to be computed.

Let s denote the number of points belonging to Z, d the integer such that $\binom{d+1}{2} \leq s < \binom{d+2}{2}$, and define

$$N(s) = \begin{cases} d + 1 - s + \binom{d+1}{2} & \text{if } \binom{d+1}{2} \leq s \leq \dfrac{d(d+2)}{2} \\ d + 2 + s - \binom{d+2}{2} & \text{if } \dfrac{d(d+2)}{2} \leq s < \binom{d+2}{2}. \end{cases}$$

Geramita and Maroscia [6] have shown that almost all sets of s points in $\mathbf{P}^2$ are defined by exactly $N(s)$ equations. We give a new proof of this fact (1.7). However, $\nu(I(Z))$ is not constant on the sets of s points in generic position. It follows from a theorem of Dubreil [4] that the best one can say is that if Z lies in generic position, then $N(s) \leq \nu(I(Z)) \leq d + 1$.