PACIFIC JOURNAL OF MATHEMATICS
Vol. 120, No. 1, 1985

# A UNIFIED APPROACH TO CARLESON MEASURES AND $A_p$ WEIGHTS. II

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**In this note we find for each $p$, $1 < p < \infty$, a necessary and sufficient condition on the pair $(\mu, v)$ (where $\mu$ is a measure on $\mathbf{R}^{n+1}_+ = \mathbf{R}^n \times [0,\infty)$, and $v$ a weight on $\mathbf{R}^n$) for the Poisson integral to be a bounded operator from $L^p(\mathbf{R}^n, v(x)\,dx)$ into $L^p(\mathbf{R}^{n+1}_+, \mu)$.**

**1. Introduction.** In this note we find for each $p$, $1 < p < \infty$, a necessary and sufficient condition on the pair $(\mu, v)$ (where $\mu$ is a measure on $\mathbf{R}^{n+1}_+ = \mathbf{R} \times [0,\infty)$ and $v$ a weight on $\mathbf{R}^n$) for the Poisson integral to be a bounded operator from $L^p(\mathbf{R}^n, v(x)\,dx)$ into $L^p(\mathbf{R}^{n+1}_+, \mu)$.

Our proof follows the ideas of Sawyer [7] and the condition we find is

$$(F_p)\qquad \int_{\tilde{Q}} \left[\mathcal{M}\left(v^{1-p'}\chi_Q\right)(x,t)\right]^p d\mu(x,t) \le C\int_Q v^{1-p'}(x)\,dx < +\infty$$

for all cubes in $\mathbf{R}^n$ (cube will always means a compact cube with sides parallel to the coordinate axes).

For $\mathcal{M}$ we denote the maximal operator

$$(*)\qquad \mathcal{M}f(x,t) = \sup_Q \frac{1}{|Q|}\int_Q |f(x)|dx, \qquad x \in \mathbf{R}^n, t \ge 0,$$

where the supremum is taken over the cubes $Q$ in $\mathbf{R}^n$, containing $x$ and having side length at least $t$.

As usual $\tilde{Q}$ denotes the cube in $\mathbf{R}^{n+1}_+$, with the cube $Q$ as its basis.

Carleson [1] showed that $\mathcal{M}$ is bounded from $L^p(\mathbf{R}^n, dx)$ into $L^p(\mathbf{R}^{n+1}_+, \mu)$ if and only if $\mu$ satisfies the so-called "Carleson condition"

$$(1)\qquad \mu(\tilde{Q}) \le C|Q| \quad \text{for each cube in } \mathbf{R}^n.$$

Afterwards, Fefferman and Stein [2] found that

$$(2)\qquad \sup_{x\in Q} \frac{\mu(\tilde{Q})}{Q} \le Cv(x) \quad \text{a.e. } x$$

is sufficient for $\mathcal{M}$ to be bounded from $L^p(\mathbf{R}^n, v(x)\,dx)$ into $L^p(\mathbf{R}^{n+1}_+, \mu)$.