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FINDING A BOUNDARY FOR A HILBERT CUBE MANIFOLD BUNDLE

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In this paper we develop an obstruction theory for the problem of determining whether a bundle, E, over a compact polyhedron, B, with non-compact Hilbert cube manifold fibers admits a boundary in the sense that there exists a compact bundle $\overline{E}$ over B with Q-manifold fibers and a sliced Z-set, $A \subset \overline{E}$, such that $\overline{E} = A \cup E$. Included in the work is a new result on fibered weak proper homotopy equivalences, a theorem on proper liftings of homotopies, and the development of a sliced shape theory whose equivalences are shown to classify our boundaries through a tie to Q-manifold theory via a sliced version of Chapman's Complement Theorem.

1. Introduction. Browder, Levine, and Livesay, [3], and Siebenmann, [26], studied the question of putting a boundary on a finite dimensional manifold. Adopting a somewhat more general, Z-set notion of a boundary, Chapman and Siebenmann successfully obtained an obstruction theory for the problem of determining which Q-manifolds admit boundaries in [14]. Interest has since been expressed in the boundary problem in a (locally trivial) bundle setting (see the problem section in the back of [8] and, more recently, problem (QM 10) in [20].) The purpose of this paper is to investigate the question of when a bundle with non-compact Q-manifold fibers can be compactified in such a way that the resulting manifold has a bundle structure extending the original bundle structure. In another variation on the boundary problem Chapman, [9], has developed machinery for deciding if a Q-manifold admits a controlled boundary. The boundary is controlled in the sense that one is given an arbitrary map from the non-compact manifold to a parameter space and desires to find a compactification along with an extension of the given map. The controlled boundary problem is evidently more general than the question studied in this paper and it would be interesting to investigate the relationship between the results of the two theories.

We begin with the introduction of some terminology and notation. All spaces except function spaces will be locally compact, separable metric spaces. A (locally trivial) bundle, $M \xrightarrow{p} B$, will be called a *Q-manifold bundle* if its fiber, F, is a Q-manifold, i.e. a manifold modeled on the