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A COEFFICIENT INEQUALITY FOR FUNCTIONS OF POSITIVE REAL PART WITH AN APPLICATION TO MULTIVALENT FUNCTIONS

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We obtain sharp bounds on the magnitude of certain determinants, whose entries are the coefficients of a function of positive real part in the unit disk. These inequalities are used to solve a coefficient problem for a certain subclass of multivalent functions.

Introduction. Let $P(z) = c_0 + c_1 z + \cdots$ be analytic in $\Delta = \{z: |z| < 1\}$ and satisfy $\operatorname{Re}(P(z)) > 0$ for z in Δ. The author **[6]** proved that the coefficients satisfy the inequality $|c_n/c_0 - c_1 c_{n-1}/c_0^2| \leq 2$ for all $n \geq 2$. This inequality was then used to obtain sharp bounds on the coefficients of functions in a subclass of multivalent close-to-convex functions. The inequality has recently been used by Libera and Zlotkiewicz **[4]** in their study of the coefficients of the inverses of convex functions. In §2 of this paper we generalize the above inequality, obtaining precise bounds on the magnitude of certain determinants involving the coefficients c_n.

Section 3 of the paper deals with the coefficient problem for a certain class of multivalent functions. Goodman **[1]** has conjectured that if $f(z) = a_1 z + \cdots$ is at most p-valent in Δ then for $n \geq p + 1$,

$$(1.1) \qquad |a_n| \leq \sum_{t=1}^{p} \frac{2t(n+p)!}{(p+t)!(p-t)!(n-p-1)!(n^2-t^2)} |a_t|.$$

Inequality (1.1) reduces to the well-known Bieberbach conjecture when $p = 1$. Let $S(p)$ be the class of functions which are analytic and p-valently starlike in Δ. A function $f(z)$, analytic in Δ with $f(0) = 0$, is a member of $S(p)$ if and only if there exists $\delta > 0$ such that for $\delta < |z| < 1$,

$$(1.2) \qquad \operatorname{Re}\left[\frac{zf'(z)}{f(z)}\right] > 0$$

and for $\delta < r < 1$,

$$(1.3) \qquad \int_0^{2\pi} \operatorname{Re}\left[\frac{re^{i\theta} f'(re^{i\theta})}{f(re^{i\theta})}\right] d\theta = 2p\pi.$$