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FREE PRODUCTS OF TOPOLOGICAL GROUPS WITH AMALGAMATION. II

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The fundamental problem is to determine if the free product with amalgamation of Hausdorff topological groups exists and is Hausdorff. This is known to be true if the subgroup being amalgamated is central or if all groups concerned are k_ω and the amalgamation subgroup is compact. In this paper a general result is proved which allows one to move outside the class of compact or central amalgamations. Using this result it follows, for example, that the amalgamated free product $F *_A G$ exists and is Hausdorff if F, G and A are k_ω-groups and A is the product of a central subgroup and a compact subgroup.

1. Introduction. The fundamental problem in this subject is to prove that the free product with amalgamation of any Hausdorff topological group exists, is Hausdorff and its underlying group structure is the amalgamated free product of the underlying groups. There have been three contributions to this problem. The first was by Ordman [9] who settled the problem for some locally invariant Hausdorff groups. The case when the amalgamated subgroup is central was settled in Khan and Morris [2]. In Katz and Morris [1] the first step was made towards handling the important class of k_ω-groups. There the case where the groups are k_ω-spaces and the amalgamated subgroup is compact is dealt with. In this paper we show that the condition that A be compact can be weakened.

We denote by $F *_C G$ the free product of the topological groups F and G with the common subgroup C amalgamated. Given k_ω-groups F and G and a common subgroup C we define the notion of the triple $(F, G;\ C)$ being beseder. If $(F, G;\ C)$ is beseder then it is readily seen that $F *_C G$ exists, is Hausdorff and has the appropriate algebraic structure. The main theorem says that $(F, G;\ C)$ is beseder if $C = A \cdot B$ where $(F, G;\ A)$ and $(F, G;\ B)$ are beseder and A is compact. So this theorem provides a procedure for progressively enlarging the family of known beseder triples.

In Katz and Morris [1] it is proved that if A is a compact subgroup of the k_ω-groups F and G then $(F, G;\ A)$ is beseder. Using results of Khan and Morris [2, 3] we prove here that if B is a closed central subgroup of the k_ω-groups F and G then the triple $(F, G;\ B)$ is beseder. Thus we can