PACIFIC JOURNAL OF MATHEMATICS
Vol. 120, No. 1, 1985

APPROXIMATE SOLUTIONS OF NONLINEAR RANDOM OPERATOR EQUATIONS: CONVERGENCE IN DISTRIBUTION

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For nonlinear random operator equations where the distributions of the stochastic inputs are approximated by sequences of random variables converging in distribution and where the underlying deterministic equations are simultaneously approximated, we prove a result about tightness and convergence in distribution of the approximate solutions. We apply our result to a random differential equation under Peano conditions and to a random Hammerstein integral equation and its quadrature approximations.

1. Introduction. In [**15**], we developed a theory of convergence of approximate solutions of random operator equations using concepts like consistency, stability, and compactness in sets of measurable functions. The results of that paper are valid for rather general notions of convergence including almost-sure convergence and convergence in probability, but excluding convergence in distribution. Of course, all the results in [**15**] that guarantee e.g. almost-sure convergence of approximate solutions imply their convergence in distribution. However, an adequate theory for convergence in distribution should also use weaker assumptions on the way the "stochastic inputs" (operator, right-hand side) are approximated that do not imply e.g. almost-sure convergence of the "stochastic outputs" (approximate solutions). It is shown in the concluding remarks of [**15**] that it is not possible to carry over the theory developed there to the case of convergence in distribution in a straightforward way.

In this paper, we prove a result about convergence in distribution of approximate solutions of random operator equations in fixed-point form; the conditions needed are such that they do not imply stronger modes of convergence for the approximate solutions: The stochastic quantities entering into the equation are approximated with respect to convergence in distribution only. Note that convergence in distribution is often sufficient for approximating statistical characteristics of the solution, since if (x_n) converges to x in distribution, then $(E(f(x_n))) \to E(f(x))$ for all bounded continuous real functions f, where E denotes the expected value (see [**6**, p. 23]).