PACIFIC JOURNAL OF MATHEMATICS
Vol. 120, No. 1, 1985

TRIDIAGONAL MATRIX REPRESENTATIONS OF CYCLIC SELF-ADJOINT OPERATORS. II

J. DOMBROWSKI

A bounded cyclic self-adjoint operator C defined on a separable Hilbert space H can be represented as a tridiagonal matrix with respect to the basis generated by the cyclic vector. An operator J can then be defined so that $CJ - JC = -2iK$ where K also has tridiagonal form. If the subdiagonal elements of C converge to a non-zero limit and if K is of trace class then C must have an absolutely continuous part.

1. Introduction. A bounded cyclic self-adjoint operator C defined on a separable Hilbert space H has a tridiagonal matrix representation with respect to the basis generated by the cyclic vector. The spectral properties of C are studied in **[1]** under the assumption that the main diagonal elements in the tridiagonal representation are zeros. In this case C is the real part of a weighted shift operator, and if J is the corresponding imaginary part then $CJ - JC = 2iK$ where K is diagonal. It is shown in **[1]** that C has an absolutely continuous part if K is of trace class.

The purpose of this paper is to extend the above result to tridiagonal matrices with non-zero diagonal. This extension is significant for the study of systems of orthogonal polynomials which satisfy a three term recursion formula. The coefficients in the recursion formula correspond to a unique tridiagonal matrix whose spectral measure is the measure of orthogonality for the system of polynomials. If the measure is symmetric about the origin (as in the case of the normalized Legendre polynomials) then the diagonal entries of the corresponding tridiagonal matrix will be zero. See **[1]** for further background.

Suppose now that C is a bounded cyclic self-adjoint operator with the following tridiagonal matrix representation with respect to the basis $\{\phi_n\}$:

$$(1.1) \qquad C = \begin{bmatrix} b_1 & a_1 & 0 & 0 & \cdots \\ a_1 & b_2 & a_2 & 0 & \cdots \\ 0 & a_2 & b_3 & a_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix}, \qquad a_n > 0.$$

Then $\phi_n = P_n(C)\phi_1$ with $P_1(\lambda) = 1$, $P_2(\lambda) = (\lambda - b_1)/a_1$ and for $n > 2$,

$$P_n(\lambda) = \frac{(\lambda - b_{n-1})P_{n-1}(\lambda) - a_{n-2}P_{n-2}(\lambda)}{a_{n-1}}.$$