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A KRASNOSEL'SKII-TYPE THEOREM FOR UNIONS OF TWO STARSHAPED SETS IN THE PLANE

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Let S be a simply connected polygonal region in the plane, symmetric with respect to the x and y axes, such that each edge of S is parallel to one of these axes. Assume that for every set E consisting of 6 or fewer edges of S there exist points t_1 and t_2 collinear with the origin (and depending on E) such that every point in $\bigcup\{e\colon e \text{ in } E\}$ is visible via S from t_1 or t_2 (or both). Then S is a union of two starshaped sets. The number 6 is best possible.

Furthermore, an example reveals that there is no finite Krasnosel'skii number which characterizes arbitrary unions of two or more starshaped sets in the plane.

1. Introduction. We begin with some preliminary definitions. Let S be a set in R^d. For points x and y in S, we say x *sees* y via S (x is *visible* from y via S) if and only if the corresponding segment $[x, y]$ lies in S. Point x is *clearly visible* from y via S if and only if there is some neighborhood N of x such that y sees each point of $N \cap S$ via S. Set S is *starshaped* if and only if there is some point p in S such that p sees each point of S via S, and the set of all such points p is called the (convex) kernel of S.

A well-known theorem of Krasnosel'skii [**5**] states that if S is a nonempty compact set in R^d, then S is starshaped if and only if every $d + 1$ points of S are visible via S from a common point. Further, points of S may be replaced by boundary points of S to produce a stronger result. An interesting problem related to this concerns obtaining a Krasnosel'skii-type theorem for unions of starshaped sets in the plane R^2. This kind of problem is mentioned in [**10**, Prob. 6.6, p. 178] and in [**1**]. Moreover, using work by Lawrence, Hare, and Kenelly [**7**] concerning unions of convex sets, the following Krasnosel'skii-type results for unions of starshaped sets are obtained in [**2**]: (1) For S compact in R^2, S is a union of two starshaped sets if for every finite set F in the boundary of S there exist points s and t (depending on F) such that each point of F is clearly visible via S from at least one of s or t. If in addition set S is simply connected, then 'clearly visible' may be replaced by the weaker