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A NOTE ON LOCALLY A-PROJECTIVE GROUPS

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If A is an abelian group, then a group G is locally A-projective if every finite subset of G is contained in a direct summand P of G which is isomorphic to a direct summand of $\oplus_I A$. Under the assumption that A is a torsion-free, reduced abelian group with a semi-prime, right and left Noetherian, hereditary endomorphism ring, various results on locally A-projective groups are proved that generalize structure theorems for homogeneous, separable, torsion-free abelian groups.

1. Introduction. Since the publication of Baer's paper on torsion-free abelian groups [6] in 1937, many attempts have been made to give structure theorems for classes of torsion-free abelian groups reaching beyond the case of completely decomposable groups. However, even in the case of separable torsion-free abelian groups, only the homogeneous case yields some interesting results whose proofs heavily depend on the well-known structure of subgroups of the rationals $\mathbf{Q}$. Naturally, the question arises whether the results themselves depend on the consideration of subgroups of $\mathbf{Q}$ too.

A first step in answering this question was done by Arnold and Lady in 1975. In [4], they introduced the following generalization of the class of homogeneous, completely decomposable groups. If A is a torsion-free, reduced abelian group, then a group G is A-projective if it is isomorphic to a direct summand of $\oplus_I A$. In the case that both, A and G, are torsion-free and have finite rank, Arnold and Lady were able to show that most properties of homogeneous, completely decomposable groups still hold in the more general setting that the endomorphism ring $E(A)$ of A is right hereditary. In [13], Huber and Warfield showed that under these conditions on A, the ring $E(A)$ is semi-prime, right and left Noetherian, and hereditary. Using this result, the author was able to remove the finite rank condition from Arnold's and Lady's results [1]. These results are summarized in Lemma 3.1 of this paper.

The progress made suggests the question whether a similar generalization is possible for homogeneous, separable torsion-free groups. In [5], Arnold and Murley began the discussion for torsion-free abelian groups A such that $E(A)$ is a principal ideal domain, and $E(A)/I$ is torsion for all