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DEFORMATION OF SUBMANIFOLDS OF REAL PROJECTIVE SPACE

KICHOON YANG

We prove that two surfaces in $\mathbf{RP}^3$ are projective deformations of each other (in the sense of E. Cartan) if and only if their induced projective structures are equivalent with respect to the Frenet framing. This result gives a projective generalization of molding surfaces.

0. Introduction. Equivalence problems of induced structures on submanifolds of homogeneous spaces are closely related to so called G-deformation problems. In the case of surfaces in $\mathbf{RP}^3 \cong \mathrm{PGL}(4;\mathbf{R})/G_0$ the projective deformation ($\mathrm{PGL}(4;\mathbf{R})$-deformation) problem has been investigated by E. Cartan (cf. [**2**]). In this paper we establish a relationship between the notion of projective deformation of surfaces and the notion of equivalence of induced projective structures on surfaces. Our main result is that two surfaces in $\mathbf{RP}^3$ are projective deformations of each other (in the sense of E. Cartan) if and only if their induced projective structures are equivalent with respect to the Frenet framing. (Theorem 5.3)

To better understand the geometric content of our result consider the analogous situation from classical surface theory. Take two surfaces in $\mathbf{R}^3 \cong \mathbf{E}(3)/O(3)$. Then the $\mathbf{E}(3)$-deformation problem is just the equivalence problem of induced Riemannian structures (i.e., the problem of local isometry). However, $\mathbf{E}(3)$-deformations in general do not preserve the Frenet frames (of Euclidean geometry) whereas, as we shall show, $\mathrm{PGL}(4;\mathbf{R})$-deformations do preserve the Frenet frames (of projective geometry). Requiring that an $\mathbf{E}(3)$-deformation should preserve the Frenet frames is equivalent to requiring that the corresponding local isometry preserves the lines of curvature, and the solutions to this latter problem are given by molding surfaces. (See [**1**], §5.) Our result may be thought of as a projective analog of molding surfaces.

Several features distinguish the projective differential geometry from the Euclidean differential geometry:

(i) A projective structure is a G-structure of degree two, i.e., it is a subbundle of the bundle of quadratic frames, and