PACIFIC JOURNAL OF MATHEMATICS
Vol 120, No. 2, 1985

CHARACTERIZATIONS OF (H)PI EXTENSIONS

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A generalization of I. U. Bronstein's characterization for PD extensions is given and, exploiting similar ideas, HPI extensions are characterized intrinsically.

I. Introduction. Among the most important features in topological dynamics in order to classify the minimal flows are the PI and HPI towers for homomorphisms (extensions) of minimal flows ([**EGS 75**], [**AG 77**] and [**V 77**]). The techniques involved depend on transfinite induction, on hyperspaces of transfinite degree and also on rather uncontrollable algebraic features in subgroups of the universal minimal flow.

Although this theory is elegant and powerful, it would be more satisfactory if these techniques and properties were related to some internal structure, only using (finite powers of) the flow itself or the fibered product (powers).

In [**B 77**] such an intrinsic description was given for PI extensions of metric minimal flows. We shall prove a nonmetric version in §3, and we relate this characterization to the one given in [**MN 80**] (the relativized version there of).

In §4 we shall give a similar kind of characterization for HPI extensions.

I would like to thank Ta-Sun Wu for the many stimulating and encouraging conversations.

In the following we establish some notations, definitions and facts; for more detailed discussions the reader is referred to [**G 76**] and [**Wo 82**].

A *topological transformation group* (*ttg* or *flow*) $\mathscr{X}$ is a triple $\mathscr{X} = \langle T, X, \pi \rangle$, where T is a T_2 topological group, X is a compact T_2 space with unique uniformity $\mathscr{U}_X$ and π is a jointly continuous action $\pi\colon T \times X \to X$. We shall consider ttgs for a fixed (but arbitrary) topological group T and we shall suppress the action symbol, writing the action as a (left) multiplication. A ttg will then be denoted by $\mathscr{X}$ or by its phase space X only. If $x \in X$, then Tx ($\overline{Tx}$) is the *orbit* (*closure*) of x and a subset $A \subseteq X$ is *invariant* if $Tx \subseteq A$ for every $x \in A$, i.e. $TA \subseteq A$. A ttg $\mathscr{X}$ is *minimal* if $\overline{Tx} = X$ for every $x \in X$ or, equivalently, if X does not contain proper closed invariant subsets. If $x \in X$ has a minimal orbit closure, x is called