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BICONTACTUAL REGULAR MAPS

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In this paper, we will classify all rotary maps with the property that each face meets only one or two others. We will show that all such maps *are in fact regular and that they are closed under the action of the* operators D, P, opp and H_j. We will then use this information to prove this theorem: Every non-trivial rotary map whose number of edges is a power of 2 is orientable.

DEFINITIONS AND NOTATION. The following are used in this paper as defined in [6]: map; regular; rotary; chiral; Petrie path; jth order hole; symmetry; the symmetries α, β, γ, X, R, S, T; the operators D, P, H_j; $G(M)$. The generators of $G(M)$ satisfy these relations:

$$(*) \qquad I = \alpha^2 = \beta^2 = \gamma^2 = \alpha\beta\gamma = X^2 = RX\alpha = SX\beta = TX\gamma.$$

When we give defining relations for the group of a map, we will assume relations $(*)$ as given, and not mention them explicitly. The orders of R, S, T are p, q, r, respectively.

The trivial maps. An important class of maps are the "trivial" maps illustrated in Figure 1. The map ε_k is simply an equator of the sphere divided into k edges, and its dual, $D\varepsilon_k$, is a k-paneled "beachball." The maps δ_k and $D\delta_k$ on the projective plane are derived from ε_{2k} and $D\varepsilon_{2k}$ respectively by identification of antipodal points. The map M_k is one of the canonical representations of the orientable surface of genus $[k/2]$, and M_k' may be viewed as M_{k-1} with a diameter drawn in.

These maps are regular for every k, and are distinct for $k > 2$. They are exactly those maps in which R S, or T has order 2. The notation here was chosen for brevity, and differs from that in [1] and elsewhere. The correspondence with [1] is given by this table:

	ε_k	$D\varepsilon_k$	δ_k	$D\delta_k$	M_k	M_k'
k even:	$\{k,2\}_k$	$\{2,k\}_k$	$\{2k,2\}/2$	$\{2,2k\}/2$	$\{2k,2k\}_{1,0}$	$\{k,k\}_2$
k odd:	$\{k,2\}_{2k}$	$\{2,k\}_{2k}$	$\{2k,2\}_k$	$\{2,2k\}_k$	$\{2k,k\}_2$	$\{k,2k\}_2$