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# SOME ROGERS-RAMANUJAN TYPE PARTITION THEOREMS

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**Analogous to the celebrated Rogers-Ramanujan partition theorems, we obtain four partition theorems wherein the minimal difference for 'about the first half' of the parts of a partition (arranged in non-increasing order of magnitude) is 2. For example, we prove that the number of partitions of $n$, such that the minimal difference of the 'first half of the summands' (that is, first $[(t+1)/2]$ summands in a partition into $t$ summands) of any partition is 2, equals the number of partitions $n$ into summands congruent to $\pm 1, \pm 2, \pm 5, \pm 6, \pm 8, \pm 9 \pmod{20}$.**

**1. Introduction.** Throughout this paper, $|x| < 1$ and we use the notation:

$$(x)_n = (1-x)(1-x^2)\cdots(1-x^n), \qquad n = 1, 2, \ldots;$$

$$\phi(a, x) = (1-a)(1-ax)(1-ax^2)\cdots \quad \text{to } \infty.$$

$$\phi(x) = \phi(x, x).$$

$\pi_t(n)$ denotes a partition of $n$ into $t$ parts arranged in non-increasing order of magnitude, say,

(1.1) $\qquad \pi_t(n) = n_1 + n_2 + \cdots + n_t; \qquad n_1 \geq n_2 \geq \cdots \geq n_t.$

For convenience, we shall refer to $n_1, n_2, \ldots$ as the first, second,... part in $\pi_t(n)$. The differences $n_1 - n_2, n_2 - n_3, \ldots$ are referred to as the first, second,... differences of the parts of $\pi_t(n)$. The "first half of the parts in $\pi_t(n)$" are defined to be the parts $n_1, n_2, \ldots, n_{[(t+1)/2]}$, where $[x]$ denotes the greatest integer function. Thus these parts are

$$n_1, n_2, \ldots, n_{t/2} \quad \text{if } t \text{ is even,}$$

and

$$n_1, n_2, \ldots, n_{(t+1)/2} \quad \text{if } t \text{ is odd.}$$

These are also described as the parts in the first half of the partition. We shall also have occasion to speak frequently about the minimal differences of the first half of the parts in $\pi_t(n)$. This will be the minimum of the differences

$$n_1 - n_2, n_2 - n_3, \ldots, n_{[(t-1)/2]} - n_{[(t+1)/2]},$$

the last of these being "the last difference in the first half of the summands of $\pi_t(n)$".