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EXISTENCE OF STRONG SOLUTIONS TO SINGULAR NONLINEAR EVOLUTION EQUATIONS

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The nonlinear evolution equation $u'(t) + A(t)u(t) \ni 0$ is studied under conditions which permit $A(t)$ to be singular at $t = 0$. Application is then made to examples of partial differential equations having time dependent coefficients which blow up at the origin.

1. Introduction. We are concerned with gaining basic insights into existence of strong solution to the abstract Cauchy problem

$$(\mathrm{ACP})_s \qquad \frac{du}{dt} + A(t)u(t) \ni 0, \qquad 0 \le s \le t \le T,$$

$$u(s) = x,$$

when hypotheses are imposed on $A(t)$ which are weak enough to allow for singularity at $t = 0$. Here u: $[s, T] \to X$, where X is an arbitrary Banach space. The operators $\{A(t)\}_{s \le t \le T}$ are assumed to satisfy

(A.0) For a.e. t in $[0, T]$, $A(t)$ is a nonlinear, possibly multivalued operator on X,

(A.1) There exists $\overline{D} \subseteq X$ such that $\overline{\mathrm{Dom}\, A(t)} \equiv \overline{D}$ for a.e. t,

and in addition, the m-accretive type conditions: for some real number ω and for λ_0 satisfying $\omega\lambda_0 < 1$,

(A.2) $\mathrm{Ran}(I + \lambda A(t)) \supseteq \overline{D}$ for a.e. t and $0 < \lambda < \lambda_0$,

(A.3) For a.e. t, the resolvent operator $J_\lambda(t) \equiv [I + \lambda A(t)]^{-1}$ exists as a Lipschitz mapping on $\overline{D}$ with $\langle J_\lambda(t)\rangle_{\mathrm{Lip}} \le (1 - \omega\lambda)^{-1}$.

Now $u(t)$ shall be referred to as a *strong solution* to $(\mathrm{ACP})_s$ if either

(S.1)
- (i) $u(t)$ is continuous on $[s, T]$ and $u(s) = x$,
- (ii) $u(t)$ is differentiable almost everywhere and satisfies the differential equation of $(\mathrm{ACP})_s$ a.e.,
- (iii) $u(t)$ is absolutely continuous on $[s, T]$

or

(S.2)
- (i) and (ii) as above and
- (iii)′ $u(t)$ is absolutely continuous on compact subsets of (s, T).