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ON BANACH SPACES HAVING A RADON-NIKODYM DUAL

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The purpose of this paper is to prove a new characterisation of Banach spaces having a Radon-Nikodym dual, namely that if E is a Banach space, then E' has the Radon-Nikodym property if and only if there exists an equivalent norm on E such that for each E-valued measure m of bounded variation, there exists an E'-valued function f with norm 1 $|m|$-a.e. such that $|m| =$ f.m.

1. Introduction. In [**1**], we have proved that if E is a Banach space, m an E-valued measure defined on a σ-algebra $\mathscr{A}$ of subsets of a set T, with bounded variation $|m|$, and if ε is any positive number, then there exists an E'-valued strongly measurable function f defined on the set T, such that $\|f\| < 1 + \varepsilon$ and $|m|(A) = \int_A f\,dm$ for each A in $\mathscr{A}$.

A very natural question which arises is the following: Does there always exist an E'-valued strongly measurable function with norm 1 such that $|m|(A) = \int_A f\,dm$ for each A in $\mathscr{A}$? Following the example given in [**1**], this seems to be possible.

Finally, an answer to that question was provided by F. Delbaen who proved the following unpublished theorem: If E is a Banach space, the following are equivalent:

(a) E' has the Radon-Nikodym property

(b) For each equivalent norm on E, for each E-valued measure m of bounded variation defined on a σ-algebra $\mathscr{A}$ of subsets of a set T, there exists a $|m|$-strongly measurable function f from T to E' such that $\|f\| = 1$ $|m|$-a.e. and $|m|(A) = \int_A f\,dm$ for each A in $\mathscr{A}$.

The purpose of this paper is to provide a positive answer to the following question: Is it possible to weaken assertion (b) by requiring the existence of an equivalent norm on the space having the property instead of assuming it for each equivalent norm on E.

2. Proof of the theorem. Before proving our theorem let us recall the Mazur density theorem and prove two lemmas.