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THE ISOTROPY REPRESENTATION FOR HOMOGENEOUS SIEGEL DOMAINS

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This paper gives several new characterizations of symmetric domains among the class of homogeneous Siegel domains. These characterizations involve the commutativity of the algebra of invariant differential operators, the transitivity of the action of the isotropy group on the Šilov boundary, and the representation of the almost complex structure by the infinitesimal isotropy action, respectively.

Homogeneous Siegel domains (equivalently, homogeneous bounded domains) are important geometric objects to study. They are more general than Hermitian symmetric spaces. They form a special class of homogeneous Kähler manifolds which would be one of the three building blocks of an arbitrary homogeneous Kähler manifold according to a conjecture of Gindikin and Vinberg. But they have also certain properties which are typical for arbitrary homogeneous Riemannian manifolds of non-positive curvature (NC algebras).

In this paper we investigate certain relations between homogeneous Siegel domains and the three different types of homogeneous spaces mentioned above. In particular, as the main result of this paper we give five different new characterizations of symmetric domains amongst the class of homogeneous Siegel domains. So if D is a homogeneous Siegel domain, G the identity component of the automorphism group of D, and K the isotropy subgroup at a point b of D, then the following are equivalent:

(a) D is symmetric

(b) The almost complex structure map on the tangent space T_bD at b is in the image of the infinitesimal isotropy representation.

(c) There exist no nontrivial G invariant vector fields

(d) The algebra of G invariant differential operators on D is commutative

(e) The isotropy group acts transitively on the Šilov boundary

(f) There exists a vector field X in the center of the isotropy algebra at the basepoint b of $D \subset \mathbf{C}^n$ so that the differential of X is invertible as a linear transformation on $\mathbf{C}^n$.