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QUASI-NORMAL STRUCTURES FOR CERTAIN SPACES OF OPERATORS ON A HILBERT SPACE

ANTHONY TO-MING LAU AND PETER F. MAH

Let E be a dual Banach space. E is said to have quasi-weak*-normal structure if for each weak* compact convex subset K of E there exists $x \in K$ such that $\|x - y\| < \operatorname{diam}(K)$ for all $y \in K$. E is said to satisfy Lim's condition if whenever $\{x_\alpha\}$ is a bounded net in E converging to 0 in the weak* topology and $\lim \|x_\alpha\| = s$ then $\lim_\alpha \|x_\alpha + y\| = s + \|y\|$ for any $y \in E$. Lim's condition implies (quasi) weak*-normal structure. Let H be a Hilbert space. In this paper, we prove that $\mathscr{T}(H)$, the space of trace class operators on H, always has quasi-weak*-normal structure for any H; $\mathscr{T}(H)$ satisfies Lim's condition if and only if H is finite dimensional. We also prove that the space of bounded linear operator on H has quasi-weak*-normal structure if and only if H is finite dimensional; the space of compact operators on H has quasi-weak-normal structure if and only if H is separable. Finally we prove that if X is a locally compact Hausdorff space, then $C_0(X)^*$ satisfies Lim's condition if and only if $C_0(X)^*$ is isometrically isomorphic to $l_1(\Gamma)$ for some non-empty set Γ.

1. Introduction. Let E be a Banach space. A bounded convex subset K of E has *normal structure* if every non-trivial convex subset H of K contains a point x_0 such that

$$\sup\{\|x_0 - y\| : y \in H\} < \operatorname{diam}(H).$$

Here $\operatorname{diam}(H) = \sup\{\|x - y\| : x, y \in H\}$ denotes the diameter of H. The Banach space E is said to have normal structure if every bounded closed convex subset of E has normal structure. If E is a dual space then E is said to have weak* normal structure if every weak* compact convex subset of E has normal structure. In **[6]** Lim introduced the notion of weak* normal structure and proved that l_1 has this property. It also follows from the proof of Theorem 3 in **[4]** that $l_1(\Gamma)$ has the same property for any non-empty set Γ. Furthermore, an application of Proposition 2 in **[9]** shows that $l_\infty(\Gamma)$ has weak* normal structure if and only if Γ is a finite set.

Let H be a Hilbert space. Let $\mathscr{B}(H)$ be the space of bounded linear operators from H into itself with the operator norm. Let $\mathscr{C}(H)$ be the closed ideal of compact operators in $\mathscr{B}(H)$. Then, as is well known,