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ON THE DUNFORD-PETTIS PROPERTY OF FUNCTION MODULES OF ABSTRACT L-SPACES

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The main result of this note states that a function module of Banach spaces has the Dunford-Pettis Property, provided that all summands are spaces of the form $L_1(\mu)$. As a corollary we obtain that every injective Banach lattice has the Dunford-Pettis Property. Another corollary states that certain spaces of compact operators have the Dunford-Pettis Property.

1. Introduction. In 1940, Nelson Dunford and Bill Pettis published their now classical result that weakly compact operators defined on $L_1(\mu)$ are completely continuous. Ten years later Grothendieck showed that the space of real-valued continuous functions on any compact topological space enjoys the same property which today is called the Dunford-Pettis Property. Specifically:

DEFINITION. Let E be a Banach space and assume that every weakly compact operator $T: E \to F$, F a Banach space, sends weakly compact subsets of E into norm compact subsets of F. Then we say that E has the Dunford-Pettis Property.

Since the early 50's the Dunford-Pettis Property has attracted much attention in the theory of Banach spaces (see the survey article of J. Diestel [5] for the historical development). However, up to today it is not quite clear which Banach spaces have the Dunford-Pettis Property. Until one or two years ago, even the following question was unanswered:

Question. If X is a compact Hausdorff space and if E is a Banach space with the Dunford-Pettis Property, does the space of all vector valued continuous functions $C(X, E)$ also have the Dunford-Pettis Property?