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METRICALLY INVARIANT MEASURES ON LOCALLY HOMOGENEOUS SPACES AND HYPERSPACES

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We compare different invariance concepts for a Borel measure μ on a metric space. μ is called open-invariant if open isometric sets have equal measure, metrically invariant if isometric Borel sets have equal measure, and strongly invariant if any non-expansive image of A has measuure $\leq \mu(A)$. On common hyperspaces of compact and compact convex sets there are no metrically invariant measures. A locally compact metric space is called locally homogeneous if any two points have isometric neighbourhoods, the isometry transforming one point into the other. On such a space there is a unique open-invariant measure, and this measure is even strongly invariant.

1. Introduction. There are two important classes of spaces with a "natural volume function": locally compact groups with Haar measure and Riemannian manifolds with their volume form. Since in everyday life volume of sets is calculated from length measurements, we consider measures invariants with respect to a metric structure rather than a group structure or differentiable structure. We deal with Borel measures on locally compact metric spaces which are finite on compact sets and metrically invariant in the sense that

"congruent sets have equal measure".

Two sets in a metric space are congruent if there is an isometry f from one onto the other. In Euclidean R^n such f can be extended to an isometry $\bar{f}$ from the whole space onto itself but in general this is not the case (cf. Example 2).

If on a metric space (X, d) there is a unique (up to a constant factor) metrically invariant measure, it can be considered as the "natural volume function" of the space. This is the case for locally compact metric groups with left-invariant metric **[2]**. The volume form on Riemannian manifolds is metrically invariant with respect to the interior metric **[18]**, and the volume form on manifolds in R^n is invariant with respect to the Euclidean metric **[10]**. However, in general it is not uniquely determined by this property (cf. Example 1).