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ON THE RADIAL MAXIMAL FUNCTION OF DISTRIBUTIONS

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We show that if the radial maximal function of a distribution $f \in \mathscr{D}(R^n)'$ belongs to $L^p(R^n)$, then f belongs to $H^p(R^n)$. This gives an affirmative answer to the question posed by Aleksandrov and Havin.

1. Introduction. Functions and distributions considered are real-valued. For $x = (x_1, \dots, x_n) \in R^n$, $t > 0$, $a > 0$ and for a measurable function $h(x)$ defined on R^n let

$$|x| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2},$$

$$B(x,t) = \{y \in R^n : |x - y| < t\},$$

$$Q(x,t) = \{(y,s) : y \in B(x,t),\ s \in (0,t)\},$$

$$(h)_t(x) = t^{-n} h(x/t)$$

and

$$\|h\|_{\Lambda_a} = \sup_{x \in R^n,\ t>0}\ \inf_{P:\ \deg P \le a}\ t^{-n-a} \int_{B(x,t)} |h(y) - P(y)|\, dy,$$

where the infimum is taken over all polynomials $P(y)$ of degree $\le a$. Let

$$\Lambda_a(R^n) = \left\{h \in L^1_{\text{loc}}(R^n) : \|h\|_{\Lambda_a} < +\infty\right\}.$$

For a multi-index $\alpha = (\alpha_1, \dots, \alpha_n)$, where α_i's are nonnegative integers, let

$$|\alpha| = \sum_{i=1}^{n} \alpha_i,$$

$$x^\alpha = x_1^{\alpha_1} \cdots x_n^{\alpha_n}$$

and

$$D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \cdots \partial x_n^{\alpha_n}}.$$