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THE SPECTRUM OF AN INTERPOLATED OPERATOR AND ANALYTIC MULTIVALUED FUNCTIONS

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Let $[B_0, B_1]$ be a complex interpolation pair and T: $B_0 + B_1 \to B_0 + B_1$ be a linear map whose restriction to each interpolation space $[B_0, B_1]_s$ is a bounded operator on that space with spectrum $\mathrm{Sp}_s T$. Under mild conditions on T it is shown that the set-valued map $\lambda \to \mathrm{Sp}_{(\mathrm{Re}\,\lambda)} T$ is an analytic multivalued function. This fact is used to unify and generalise a number of previously known results about the spectrum of an interpolated operator, and also to motivate some new ones.

Introduction. Let $[B_0, B_1]$ be a complex interpolation pair and $B_s = [B_0, B_1]_s$ $(0 \le s \le 1)$ be the corresponding interpolation spaces. If T: $B_0 + B_1 \to B_0 + B_1$ is a linear map whose restriction to each B_s is a bounded operator on B_s, then its spectrum $\mathrm{Sp}_s T$ in $L(B_s)$ can vary with s. This phenomenon has been investigated in a wide variety of special cases. Examples include: certain sorts of matrices on l_p-spaces [**15, 16, 27, 47**], Cesaro-type operators on L_p-spaces [**10, 26, 35**], multipliers on the L_p-spaces of a locally compact Abelian group [**28, 36, 50, 51, 52**], and certain singular integral operators on L_p-spaces [**29**], and even H_p-spaces [**14**]. The main theoretical results have been of three kinds: conditions ensuring that $\mathrm{Sp}_s T$ is independent of s [**17, 21, 22, 36, 48**], establishment of bounds for $\mathrm{Sp}_s T$ in terms of $\mathrm{Sp}_0 T$ and $\mathrm{Sp}_1 T$ [**46, 48**], and investigation of the continuity properties of the set-valued map $s \to \mathrm{Sp}_s T$ [**43, 44, 45, 52**].

This last is the starting point for the present paper. Although the upper semicontinuity of $s \to \mathrm{Sp}_s T$ for $s \in (0,1)$ is a purely topological statement, its proof in [**44**] depends upon properties of analytic functions. This state of affairs seems somewhat unsatisfactory: surely from such a proof it should be possible to draw *analytic* conclusions. We do just that, using the recently developed tools from the theory of analytic multivalued functions. This theory was first applied by Z. Słodkowski in [**38**] to describe the spectrum of an analytically varying operator on a Banach space; by contrast we keep the operator fixed and allow the space to vary. One of our two main results (2.7) asserts that the map $\lambda \to \mathrm{Sp}_{(\mathrm{Re}\,\lambda)} T$ is an analytic multivalued function on $\{\lambda;\ 0 < \mathrm{Re}\,\lambda < 1\}$. Unfortunately, technical problems force us to impose a mild condition on T for the proof to