PACIFIC JOURNAL OF MATHEMATICS
Vol. 121, No. 2, 1986

VECTOR BUNDLES OVER $(8k+3)$-DIMENSIONAL MANIFOLDS

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Let M be a closed, connected, smooth and 3-connected mod 2 (i.e. $H_i(M;\mathbf{Z}_2)=0$, $0<i\le 3$) manifold of dimension $3+8k$ with $k>1$. We obtain some necessary and sufficient condition for the span of a $(3+8k)$-plane bundle η over M to be greater than or equal to 7 or 8. We obtain, for M 4-connected mod 2 and satisfying $\mathrm{Sq}^2\,\mathrm{Sq}^1 H^{n-8}(M)=\mathrm{Sq}^2 H^{n-7}(M)$, where $n=\dim M\equiv 11 \mod 16$ with $n>11$, that span $M\ge 8$ if and only if $\chi_2(M)=0$. Some applications to product manifolds and immersion are given.

1. Introduction. Let M be a closed, connected and smooth manifold whose dimension n is congruent to 3 mod 8 with $n\ge 11$. Let η be an n-plane bundle over M. Recall span(η) is defined to be the maximal number of linearly independent cross sections of η. When η is the tangent bundle of M we simply write span(M) for span(η). Recall that the Kervaire mod 2 semi-characteristic of M $\chi_2(M)$, is defined by

$$\chi_2(M)=\sum_{2i<n}\dim_{\mathbf{Z}_2} H^i(M;\mathbf{Z}_2) \mod 2.$$

Suppose M is a 1-connected mod 2 spin manifold. Then according to Thomas [14] and Randall [11], span$(M)\ge 4$ if and only if $\chi_2(M)=0$ and $w_{n-3}(M)=0$, where $w_j(M)$ is the jth-mod 2 Stiefel-Whitney class of the tangent bundle of M. In this paper we shall derive some necessary and sufficient condition for span$(M)\ge 7$ or 8 when M is 3-connected mod 2.

For the rest of the paper we shall assume that M is 3-connected mod 2. Then from [14] and the methods of Mahowald [4] we have:

THEOREM 1.1 (*Thomas-Mahowald*). Span$(M)\ge 5$ *if and only if* $\delta w_{n-5}(M)=0$ *and* $\chi_2(M)=0$. *Here δ is the Bockstein coboundary homomorphism associated with the exact sequence* $0\to\mathbf{Z}\to\mathbf{Z}\to\mathbf{Z}_2\to 0$.

We shall consider the modified Postnikov tower for the fibration $B\mathrm{spin}_{n-k}\to B\mathrm{spin}_n$ for $n\ge 19$ and $k=7$ or 8 where $B\mathrm{spin}_j$ is the classifying space of spin j-plane bundles. Then for an n-plane bundle η over M, η is classified by a map g from M into $B\mathrm{spin}_n$. Then η admits k-linearly independent cross-sections if and only if g lifts to $B\mathrm{spin}_{n-k}$.