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THE BOUNDARY REGULARITY OF THE SOLUTION OF THE $\bar{\partial}$-EQUATION IN THE PRODUCT OF STRICTLY PSEUDOCONVEX DOMAINS

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Let D be a strictly pseudoconvex domain in $\mathbf{C}^n$. We prove that for every $\bar{\partial}$-closed differential $(0, q)$-form f, $q \geq 1$, with coefficients of class $\mathscr{C}^\infty(D \times D)$, and continuous in the set $\bar{D} \times \bar{D} \setminus \Delta(D)$, the equation $\bar{\partial} u = f$ admits a solution u with the same boundary regularity properties. As an application, we prove that certain ideals of analytic functions in strictly pseudoconvex domains are finitely generated.

1. Introduction. Let D be a bounded strictly pseudoconvex domain in $\mathbf{C}^n$ with $\mathscr{C}^2$ boundary. It is known ([**2**], Theorem 2) that given a $(0, q)$-form f in D with coefficients of class $\mathscr{C}^\infty(D \times D)$ and continuous in $\bar{D} \times \bar{D}$, such that $\bar{\partial} f = 0$, $q = 1, \ldots, 2n$, there exists a $(0, q - 1)$-form u in $D \times D$ such that the coefficients of u are also of class $\mathscr{C}^\infty(D \times D)$ and continuous in $\bar{D} \times \bar{D}$, and such that $\bar{\partial} u = f$.

In this paper, using the results from [**2**], and the method of [**6**], we prove the following theorem:

THEOREM 1. *Let D be a bounded strictly pseudoconvex domain in $\mathbf{C}^n$ with $\mathscr{C}^2$ boundary. Set $Q = (\bar{D} \times \bar{D}) \setminus \{(z, z) | z \in \partial D\}$. Suppose that f is a $(0, q)$ $\bar{\partial}$-closed differential form with coefficients in $\mathscr{C}^\infty(D \times D) \cap \mathscr{C}(Q)$. Then there exists a $(0, q - 1)$-form u with coefficients in $\mathscr{C}^\infty(D \times D) \cap \mathscr{C}(Q)$, such that $\bar{\partial} u = f$.*

As an application, we prove a following theorem on the existence of the decomposition operators in some spaces of holomorphic functions in the product domain $D \times D$: Let D and Q be as above. Denote by $A_Q(D \times D)$ the space of all functions holomorphic in $D \times D$, which are continuous in Q. Let $(A_Q)_0(D \times D)$ be the subspace of $A_Q(D \times D)$, consisting of all functions which vanish on $\Delta(D)$, the diagonal in $D \times D$.

THEOREM 2. *Let $g_1, \ldots, g_N \in (A_Q)_0(D \times D)$ satisfy the following properties*: (i) $\{(z, s) \in Q | g_1(z, s) = \cdots = g_N(z, s) = 0\} = \Delta(D)$; (ii) for every $z \in D$, the germs at (z, z) of the functions g_i, $i = 1, \ldots, N$, generate