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# INVERSE THEOREMS FOR MULTIDIMENSIONAL BERNSTEIN OPERATORS

Z. DITZIAN

**Let $B_n f$ be the $m$-dimensional Bernstein polynomials on a simplex or on a cube. The class of functions for which $\|B_n f - f\| = O(n^{-\alpha})$ is determined. That is, necessary and sufficient conditions on the smoothness of $f$ in the simplex or the cube and especially near their boundaries are given so that $\|B_n f - f\| = O(n^{-\alpha})$. Interpolation of spaces, and in particular the characterization of the interpolation space, is one of the tools used.**

For a sequence of approximation operators an inverse theorem is a result determining necessary and sufficient conditions on the rate of convergence for the function to belong to a certain class of functions generally satisfying some smoothness conditions. A more restrictive view is that which calls the necessary and the sufficient conditions above direct and inverse theorems respectively. Here the inverse results will be of the first variety.

The Bernstein polynomials on $C[0,1]$ are given by

$$(1.1)\quad B_n(f,x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right) P_{n,k}(x) \qquad \text{where } P_{n,k}(x) \equiv \binom{n}{k} x^k (1-x)^{n-k}.$$

For $B_n(f,x)$ it was shown by Berens and Lorentz [**1**] that

$$|B_n(f,x) - f(x)| \le M\left(\frac{(x(1-x))}{n}\right)^{\alpha/2} \quad \text{for } 0 < \alpha < 2$$

occurs if and only if

$$|\Delta_h^2 f(x)| \equiv |f(x-h) - 2f(x) + f(x+h)| \le Mh^\alpha \qquad \text{for } [x-h, x+h] \subset [0,1].$$

The Bernstein polynomial on the simplex

$$S \equiv \left\{(x_1, \dots, x_m);\ x_i \ge 0,\ \sum_{i=1}^{m} x_i \le 1\right\},$$