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MINIMAL NON-PERMUTATIVE PSEUDOVARIETIES OF SEMIGROUPS. II

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This paper is the continuation of a previous paper in which all minimal non-$\mathscr{P}$-permutative pseudovarieties of $\mathscr{C}$ were determined where $\mathscr{P}$-permutativity was one of several conditions implying permutativity and $\mathscr{C}$ was the class of either (finite) groups, monoids or semigroups. In this work, the most general case of this type is treated, namely when $\mathscr{P}$-permutativity is permutativity and $\mathscr{C}$ is the class of all finite semigroups.

The notations and conventions adopted in this paper were introduced in [**1**].

1. Introduction. This paper is the continuation of [**1**] and uses the notation and conventions introduced there. In [**1**] we determined all minimal non-$\mathscr{P}$-permutative pseudovarieties of $\mathscr{C}$ where $\mathscr{P}$-permutativity was one of several conditions implying permutativity and $\mathscr{C}$ was the class of either (finite) groups, monoids or semigroups. Here, we treat the most general case of this type, namely when $\mathscr{P}$-permutativity is permutativity and $\mathscr{C}$ is the class of all finite semigroups.

2. Some non-permutative semigroups. In this section, we introduce some semigroups which will play an important role in the sequel. We also indicate a finite basis of identities for each of them.

Recall that any completely simple semigroup is isomorphic to a Rees $I \times \Lambda$ matrix semigroup $\mathscr{M}(G;\ I, \Lambda;\ P)$ over a group G with sandwich matrix $P = (p_{\lambda i})$ (see Clifford and Preston [3]). For a prime number p, we let

$$K_p = \mathscr{M}\left(\mathbf{Z}_p;\ 2, 2;\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right).$$

Rasin [5] has given a complete description of the lattice of varieties of completely simple semigroups over abelian groups. The appropriate universal algebraic type to deal with arbitrary completely simple semigroups involves not only one binary operation (product), but also one unary operation (inversion within the group containing a given element). However, in the context of finite semigroups, the need for the unary operation disappears, since the (group) inverse of an element is then one of its (positive) powers.