

MINIMAL NON-PERMUTATIVE PSEUDOVARITIES OF SEMIGROUPS. I

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A semigroup is permutative if it satisfies an identity of the form $x_1 x_2 \cdots x_n = x_{\sigma_1} x_{\sigma_2} \cdots x_{\sigma_n}$ where σ is a non-identical permutation of $\{1, 2, \dots, n\}$. The finite permutative semigroups form a pseudovariety and permutative pseudovarieties enjoy many properties first obtained for commutative pseudovarieties. Several types of permutation identities are considered, and all pseudovarieties minimal with respect to the property of containing a finite semigroup which fails an identity of a given type are determined. This includes the cases of the commutativity identity, the general permutation identities, and the “strong (left) permutation identities”. As a preliminary, all minimal non-commutative pseudovarieties of groups and monoids are also determined.

1. Introduction. As a natural generalization of commutativity, permutation identities have been considered by several authors and shown to play an important role in various contexts (see Yamada [19, 20], Perkins [12], Pollák [14, 15], Higgins [7, 8], Almeida [1]). The non-collapse of this extended concept is peculiar to the class of semigroups, as a permutative monoid is necessarily commutative.

The problem of determining the minimal non-commutative pseudovarieties of groups was first considered by S. Oates (see Neumann [11], p. 42) although this author did not exhibit all such pseudovarieties, rather just established that they must contain a non-abelian metabelian group. As a step towards the solution of a problem in language theory, Margolis and Pin [10] then extended this result by showing that a non-commutative pseudovariety of monoids all of whose group members are abelian must contain one of three monoids which they describe. They also claim that their methods can be adjusted to yield a list of generators (up to the group case) of the minimal non-commutative pseudovarieties of semigroups. However, as we show here, their list is incomplete.

In the first part of this work, we determine explicitly all minimal non-commutative pseudovarieties of groups and semigroups. We also consider some special types of permutation identities and produce a complete list of minimal non-strongly permutative and non-strongly left permutative pseudovarieties of semigroups. This is based on the solution