PACIFIC JOURNAL OF MATHEMATICS
Vol. 122, No. 1, 1986

A NEW CLASS OF ISOCOMPACT SPACES AND RELATED RESULTS

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In the last fifteen years a large number of classes of isocompact spaces were investigated by many mathematicians. In this paper we introduce a new large class (i.e. the class of k-neat spaces) of isocompact spaces. This class contains all of the following classes: neighborhood $\mathscr{F}$-spaces, spaces satisfying property θL, weakly $[\omega_1, \infty)^r$-refinable spaces, $\delta\theta$-penetrable spaces and pure spaces. Other properties of this class are also investigated. For example we show that an ω_1-compact ω_1-neat T_1-space is a-realcompact and k-neatness is an inverse invariant of maps under some conditions. In the last section we consider compactness of isocompact spaces having a countably compact dense subset.

1. Introduction and preliminaries. A space is said to be isocompact if every closed countably compact subset is compact [2]. Since Bacon's paper [2], many isocompact classes have been found. In this paper we introduce a new large class (i.e. the class of k-neat spaces) of isocompact spaces. This class properly contains the class of neighborhood $\mathscr{F}$-spaces [7], spaces satisfying property θL [6], weakly $[\omega_1, \infty)^r$-refinable spaces [21], $\delta\theta$-penetrable spaces [4], and pure spaces [1]. By this concept we can neatly review many results in the area of isocompactness.

In the second section the definition of k-neat spaces is given for an infinite cardinal k. It is proved that every k-neat space is isocompact and every ω_1-compact ω_1-neat T_1-space is a-realcompact (i.e. closed complete). These two theorems strengthen many results in this area. Behavior of k-neat spaces under some operations is investigated in the third section, and we give an example which demonstrates that neat spaces are strictly weaker than the isocompact classes listed above. In the fourth section we consider isocompact spaces having a countably compact dense subset, and provide conditions for such a space to be compact. As one case we show that a regular T_2 isocompact space is compact if it is represented as the union of a countably compact dense subset and an almost realcompact dense subset.

In this paper all maps are assumed to be continuous.

The rest of this section is devoted to some definitions used in the following sections. We denote by $\omega(\omega_1)$ the first infinite (uncountable) cardinal and $\mathscr{P}(X)$ denotes the power set of a set X.