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THE BAIRE-CATEGORY METHOD IN SOME COMPACT EXTENSION PROBLEMS

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We characterize metrizable separable spaces X such that almost every, in the sense of Baire category, embedding h of X into the Hilbert cube I^ω provides a compact extension $\overline{h(X)}$ such that the remainder $\overline{h(X)} \setminus h(X)$ has certain dimensional property (for instance, is n-dimensional, countable-dimensional or "metrically weakly infinite-dimensional"). We obtain a characterization of metrizable separable spaces which have large transfinite dimension by means of compactifications. Two examples related to the results mentioned above are constructed.

1. Introduction. Consider the following two classes of separable metrizable spaces: the class (P_n) of spaces X with $\dim X \leq n$ and the class (P_ω) of countable-dimensional spaces (for terminology see §2). In this paper, for each of the classes (P), we characterize the class of the spaces X such that almost every, in sense of Baire category, homeomorphic embedding h of X into the Hilbert cube I^ω yields a compact extension $\overline{h(X)}$ of $h(X)$ whose remainder $\overline{h(X)} \setminus h(X)$ is in the class (P).

In contrast with the classical result in dimension theory that if X has a compact extension with dim $\leq n$ (i.e. $\dim X \leq n$, by Hurewicz theorem) then almost every embedding h: $X \to I^\omega$ provides a compact extension $\overline{h(X)}$ with $\dim \overline{h(X)} \leq n$, the existence of a compactification $\tilde{Y}$ of Y whose remainder $\tilde{Y} \setminus Y$ is in the class (P), for any of the two (P) we consider, is not enough to guarantee that almost every embedding h: $Y \to I^\omega$ provides such a compactification $\overline{h(Y)}$ for $h(Y)$.

In the case of (P_n) the characterization is simple: the class consists exactly of spaces which are unions of a compact set and an n-dimensional set (§3).

To give a characterization for (P_ω), we introduce a somewhat weaker property than the weak infinite-dimensionality (in the sense of Smirnov). In particular, spaces having large transfinite dimension have this property, which yields a characterization of spaces with tr Ind, analogous to that given by Hurewicz for small transfinite dimension (§4).