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ON THE GROWTH OF MEROMORPHIC FUNCTIONS WITH RADIALLY DISTRIBUTED ZEROS AND POLES

JOSEPH MILES

The lowest possible rate of growth of a meromorphic function f of genus q with zeros and poles restricted to a given finite set of rays through the origin is determined in terms of q and the rays carrying the zeros and poles. For $\alpha > 1$ the ratio $T(\alpha r, f)/T(r, f)$ is shown to be bounded as r tends to infinity for all such entire functions, but not for all such meromorphic functions.

1. Introduction. In this paper we are concerned with the rate of growth of the Nevanlinna characteristic of meromorphic functions whose zeros and poles are restricted to lie on a finite number of rays through the origin. We consider the relationship between the order and lower order of such functions as well as upper bounds for $T(\alpha r, f)/T(r, f)$ for $\alpha > 1$.

We first specify the class of functions that we will consider. Suppose $X = \{\theta_1, \theta_2, \dots \theta_M\}$ and $\{Y = \theta_{M+1}, \theta_{M+2}, \dots \theta_L\}$ each consist of distinct members of $[0, 2\pi)$, are not both empty, and have an empty intersection. For a nonnegative integer q, let $\mathcal{M}_q(X, Y)$ be the collection of all functions meromorphic in the complex plane with zeros z_ν and poles w_ν satisfying

$$(1.1)\qquad \text{(i)}\qquad \arg z_\nu \in X,$$

$$\text{(ii)}\qquad \arg w_\nu \in Y,$$

$$\text{(iii)}\qquad \sum_\nu \frac{1}{|z_\nu|^q} + \sum_\nu \frac{1}{|w_\nu|^q} = \infty,$$

and

$$\text{(iv)}\qquad \sum_\nu \frac{1}{|z_\nu|^{q+1}} + \sum_\nu \frac{1}{|w_\nu|^{q+1}} < \infty.$$

For $X \neq \varnothing$, let $\mathscr{E}_q(X)$ be the collection of entire functions $\mathcal{M}_q(X, \varnothing)$. We note it is immediate from (1.1iii) that $f \in \mathcal{M}_q(X, Y)$ has order $\lambda \geq q$.

Our principal result (Theorem 1) enables us to determine the minimum of the lower orders μ of $f \in \mathcal{M}_q(X, Y)$ by applying a certain criterion, essentially geometric in character, to the sets

$$(1.2)\qquad S_k = \{e^{-ik\theta_j}: 1 \leq j \leq M\} \cup \{-e^{-ik\theta_j}: M + 1 \leq j \leq L\}$$