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# DISCS IN COMPRESSION BODIES

D. D. LONG

**It has been shown in a paper by Casson and the author that there is an algorithm to determine if an automorphism of a closed, orientable surface 'compresses'. A motivation for finding such a decision procedure is to apply it to fibred knots to determine if they are 'homotopically ribbon'. This makes it important that the process is as efficient as possible. One of the aims of this paper will be to exhibit improvements in this algorithm.**

**0. Introduction.** We shall devote our attentions to the improvements that can be made in the case that the automorphism is pseudo-Anosov, that is, it preserves a pair of transverse, measured, singular foliations. Our results will then apply to hyperbolic knots. In particular, this simplication cut down drastically the computation of [**6**], in which the above methods were applied to show that the knot 14* is not homotopically ribbon. Moreover, the method sheds some light on the way a pseudo-Anosov extends over a compression body.

The techniques we use are largely combinatorial, and exploit the special nature of the disc provided by [**3**] to perform traditional 3-manifold arguments.

In §1 we introduce our notation and explain the exact nature of the problem which we seek to circumvent. We may briefly explain the contents of §2 as follows. Define a closed arc lying in a leaf of a foliation to be *regular* if it contains no singularities. Then one of the results of §2 is that if a pseudo-Anosov compresses, there is an embedded disc in the compression whose boundary is the union of two arcs, one of which is regular. Our main result, Theorem 2.4, is a generalization of this, and says that there is a disc whose boundary is nearly as combinatorially simple as it could be. The precise meaning of this is explained in §1.

In the course of proving 2.4 we prove the following lemma which is of interest in its own right.

LEMMA 2.2. *Let $\phi$: $F \to F$ be a pseudo-Anosov map which extends over a compression body $M$. Then there is an integer $t$ (which is bounded by some function depending only on the genus of $F$) and an embedded disc $D \subset M$ so that if $m$ and $n$ are any positive integers, then $\phi^{tm}(D) \cap D \cap \phi^{-tn}(D)$ can be isotoped* rel$(\partial M)$ *so as to contain no triple points.*