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NEAR ISOMETRIES OF BOCHNER L^1 AND L^∞ SPACES

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Let $(\Omega_i, \Sigma_i, \mu_i)$ be σ-finite measure spaces, $i = 1, 2$, and let E be a Hilbert space. If the Bochner spaces $L^p(\Omega_1, \Sigma_1, \mu_1, E)$ and $L^p(\Omega_2, \Sigma_2, \mu_2, E)$ are nearly isometric, for either $p = 1$ or $p = \infty$, then $L^1(\Omega_1, \Sigma_1, \mu_1, E)$ is isometric to $L^1(\Omega_2, \Sigma_2, \mu_2, E)$ and hence $L^\infty(\Omega_1, \Sigma_1, \mu_1, E)$ is isometric to $L^\infty(\Omega_2, \Sigma_2, \mu_2, E)$.

Throughout this paper the letter E will denote a Banach space which will often be taken to be Hilbert space. Interaction between elements of a Banach space and those of its dual will be denoted by $\langle \cdot , \cdot \rangle$. We will write $E_1 \cong E_2$ to indicate that the Banach spaces E_1 and E_2 are isometric.

Following Banach, [**2**, p. 242], we will call the Banach spaces E_1 and E_2 nearly isometric if $1 = \inf\{\|T\|\|T^{-1}\|\}$, where T runs through all isomorphisms of E_1 onto E_2. It is of course equivalent to suppose that $1 = \inf\{\|T\|\}$, where $\|T^{-1}\| = 1$, and hence T is a norm-increasing isomorphism of E_1 onto E_2. For if T is any continuous isomorphism of one Banach space onto another, we obtain an isomorphism $\hat{T}$ having the desired properties by defining $\hat{T}$ to be equal to $\|T^{-1}\|T$.

If (Ω, Σ, μ) is a positive measure space and E a Banach space, the Bochner spaces $L^p(\Omega, \Sigma, \mu, E)$ will be denoted by $L^p(\mu, E)$ when there is no danger of confusing the underlying measurable spaces involved, and by $L^p(\mu)$ when E is the scalar field. For the definitions and properties of these spaces we refer to [**8**].

It has been noted by Benyamini [**4**] that, as a consequence of known properties of spaces of continuous functions, if two spaces $L^p(\mu_1)$ and $L^p(\mu_2)$ are nearly isometric, for either $p = 1$ or $p = \infty$, then they are isometric. What we wish to show is that the same conclusion can be drawn for near isometries of certain Bochner spaces. We will prove the following:

THEOREM. *Let $(\Omega_i, \Sigma_i, \mu_i)$ be σ-finite measure spaces, $i = 1, 2$, and E a Hilbert space. If there exists an isomorphism T, with $\|T^{-1}\| = 1$ and $\|T\| < 3/(2\sqrt{2})$, mapping $L^p(\Omega_1, \Sigma_1, \mu_1, E)$ onto $L^p(\Omega_2, \Sigma_2, \mu_2, E)$ for either $p = 1$ or $p = \infty$, then $L^1(\Omega_1, \Sigma_1, \mu_1, E) \cong L^1(\Omega_2, \Sigma_2, \mu_2, E)$ and $L^\infty(\Omega_1, \Sigma_1, \mu_1, E) \cong L^\infty(\Omega_2, \Sigma_2, \mu_2, E)$.*