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CROSSED PRODUCT AND HEREDITARY ORDERS

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Let Λ be the crossed product order $(O_L/O_K, G, \rho)$ where L/K is a finite Galois extension of local fields with Galois group G, and ρ is a factor set with values in O_L^*. Let $\Lambda_0 = \Lambda$, and let Λ_{i+1} be the left order $O_l(\text{rad}\,\Lambda_i)$ of rad Λ_i. The chain of orders $\Lambda_0, \Lambda_1, \dots, \Lambda_s$ ends with a hereditary order Λ_s. We prove that Λ_s is the unique minimal hereditary order in $A = K\Lambda$ containing Λ, that Λ_s has e/m simple modules, each of dimension f over the residue class field $\overline{K}$ of O_K, and that $s = d - (e-1)$. Here d, e, f are the different exponent, ramification index, and inertial degree of L/K, and m is the Schur index of A.

1. Introduction. Let O_K be a complete discrete valuation ring having field of fractions K and finite residue class field $\overline{K}$. Let L be a finite Galois extension of K, with Galois group G, and let O_L be the valuation ring in L. Let ρ be a factor set on $G \times G$ with values in the units of O_L. We are interested in the crossed product order $\Lambda = (O_L/O_K, G, \rho)$ contained in the simple algebra $A = (L/K, G, \rho)$. If ρ is trivial, Auslander-Goldman [**1**] showed that Λ is a maximal order in A if and only if L/K is unramified, and Auslander-Rim [**2**] showed that Λ is hereditary if and only if L/K is tamely ramified. Williamson [**8**] extended the Auslander-Rim result to the case that ρ is any factor set. We are interested in the wild case. Benz-Zassenhaus [**3**] showed that Λ is contained in a unique minimal hereditary order in A.

We set $\Lambda_0 = \Lambda$, and define inductively

$$\Lambda_{j+1} = \{x \in A\colon x\,\text{rad}\,\Lambda_j \subseteq \text{rad}\,\Lambda_j\} = O_l(\text{rad}\,\Lambda_j).$$

Then we have the sequence of orders

$$\Lambda_0 \subsetneqq \Lambda_1 \subsetneqq \Lambda_2 \subsetneqq \cdots \subsetneqq \Lambda_s = \Lambda_{s+1}$$

for some integer s. Since $\Lambda_s = O_l(\text{rad}\,\Lambda_s)$, it follows that Λ_s is hereditary ([**6**, 39.11, 39.14]). From the theory of hereditary orders (see [**6**, 39.14]) Λ_s may be described as follows: if $A \cong M_n(D)$, the ring of $n \times n$ matrices over a division ring D, and if Δ is the unique maximal order in D, then Λ_s is the set of block matrices, with entries in Δ, where there are r diagonal blocks of size $n_i \times n_i$, and blocks above the diagonal have