PACIFIC JOURNAL OF MATHEMATICS
Vol. 123, No. 1, 1986

POSITIVE DEFINITE FUNCTIONS AND L^p CONVOLUTION OPERATORS ON AMALGAMS

MASSIMO A. PICARDELLO

Let K_i be a countable collection of compact groups, and assume that $H = \bigcap_i K_i$ is an open subgroup of K_i for every i. In this paper we consider positive definite functions and convolution operators on the amalgamated product $G = *_H K_i$, and we study their properties in relation with the notion of length of reduced words. In particular, if $\sup_i k_i < \infty$, we show that there exist unbounded approximate identities in $A(G)$, that the space of bounded convolution operators on $L_p(G)$ is the dual space of the algebra $A_p(G)$, and, under the additional assumption that H be finite, that there exist unbounded approximate identities in $A(G)$.

1. Introduction. Considerable *attention* has been devoted, in the recent literature, to positive definite functions on groups acting isometrically on homogeneous trees. The Fourier-Stieltjes algebra of the free group $\mathbf{F}_r$ with r generators, which acts isometrically on the homogeneous tree of degree $2r$, has been studied in detail in [**5, 14, 9, 6, 1, 13**]. Other free products have been considered in [**16, 17, 4**]. The class of groups acting simply transitively on a homogeneous tree has been considered in [**3**]. Every locally compact group G acting isometrically on a homogeneous or semihomogeneous tree T is isomorphic to the amalgamated product $K_1 *_H K_2$, where K_1 and K_2 are the stability subgroups of two contiguous vertices and $H = K_1 \cap K_2$ is the stability subgroup of the corresponding edge [**18**]. The subgroup H is open, and its indices in K_1 and K_2 are the homogeneity degrees of T. In particular, if the homogeneity degrees are finite, G is the amalgam of two compact groups. Some properties of positive definite functions on amalgams of two factors have been studied in [**2, 11**].

In this paper we consider amalgamated products $G = *_H G_i$, where $\{G_i,\ i \in I\}$ is any collection of locally compact groups and H is a common open subgroup. These groups act isometrically on trees with periodical homogeneity degrees (and on "polygonal graphs": see [**16**]). The homogeneity degrees are finite if and only if the factors G_i are compact; they are bounded if and only if the indices k_i of H in G_i are bounded. For groups of this type, we consider several results originally obtained for free groups in [**14, 8, 13**]. Some of our arguments are adapted