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FIXED POINT THEOREMS FOR SOME DISCONTINUOUS OPERATORS

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The purpose of this paper is to show the existence of fixed points for operators T defined on a subset K of a Banach space X and belonging to a class that the author calls $D(a, b)$ with $0 \leq a,\ b \leq 1$.

1. Introduction. Let T be a mapping of a set K into itself. An immediate question is whether some point is mapped onto itself; that is, does the equation

$$Tx = x \tag{1}$$

have a solution? If so, x is called a *fixed point* of T. This question generates a theory which began in 1912 with the work of L. E. J. Brouwer, who proved that any continuous mapping T of an n-ball into itself has a fixed point, and was followed in 1922 by S. Banach's Contraction Principle, which states that any mapping T of a complete metric space X into itself that satisfies, for some $0 < k < 1$, the inequality

$$d(Tx, Ty) \leq k\, d(x, y) \tag{2}$$

for all x and y in X, has a unique fixed point. Here d denotes the metric on X. J. Schauder [**13**], Tychonoff [**16**]. S. Lefschetz [**10**], F. Browder [**2**], W. A. Kirk [**7**], and many others have added to and generalized these basic results.

In 1969 and 1971, R. Kannan [**5**], [**6**], proved some fixed point theorems for operators T mapping a Banach space X into itself which, instead of the contraction property in (2), satisfy the condition:

$$\|Tx - Ty\| \leq \alpha\left[\|x - Tx\| + \|y - Ty\|\right], \tag{3}$$

for all x, y in X; where $0 < \alpha < 1/2$. G. Hardy and T. Rogers [**4**] generalized this result to continuous mappings T of a complete metric space X into itself that satisfy:

$$\begin{aligned} d(Tx, Ty) \leq{}& a_1 d(x, y) + a_2 d(x, Tx) + a_3 d(y, Ty) \\ &+ a_4 d(x, Ty) + a_5 d(y, Tx), \end{aligned} \tag{4}$$

for all x and y in X, where $a_i \geq 0$ and $a_1 + a_2 + a_3 + a_4 + a_5 < 1$. K. Goebel, W. A. Kirk, and T. N. Shimi [**3**], extended the last result to