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CONTINUA OF CONSTANT DISTANCES IN SPAN THEORY

A. LELEK

It is proved that, for each non-negative number β not exceeding the span of a mapping f: $X \to Y$, where X and Y are compact metric spaces, there exists a non-empty continuum $K_\beta \subset X \times X$ with identical two projections and such that the distances between $f(x)$ and $f(x')$ are all equal to β for $(x, x') \in K_\beta$. Similar results hold for other types of spans.

1. Preliminaries. All spaces are assumed to be non-empty metric spaces, and all mappings to be continuous functions. Let f: $X \to Y$ be a mapping. If X is connected, the *surjective span* $\sigma^*(f)$ of f is defined to be the least upper bound of the set of real numbers α with the following property: there exist non-empty connected sets $C_\alpha \subset X \times X$ such that $\operatorname{dist}[f(x), f(x')] \geq \alpha$ for $(x, x') \in C_\alpha$, and

$$(\sigma^*) \qquad p_1(C_\alpha) = p_2(C_\alpha) = X,$$

where p_1 and p_2 denote the standard projections of the product, that is, $p_1(x, x') = x$ and $p_2(x, x') = x'$. The *span* $\sigma(f)$, the *semispan* $\sigma_0(f)$, both for mappings f with the domains X not necessarily connected, and the *surjective semispan* $\sigma_0^*(f)$ in the case of connected domains, are defined similarly with condition (σ^*) relaxed to conditions

$$(\sigma) \qquad p_1(C_\alpha) = p_2(C_\alpha),$$

$$(\sigma_0) \qquad p_1(C_\alpha) \supset p_2(C_\alpha),$$

$$(\sigma_0^*) \qquad p_1(C_\alpha) = X,$$

respectively. The following inequalities and formulae are direct consequences of the definitions:

$$(1) \qquad 0 \leq \sigma^*(f) \leq \sigma(f) \leq \sigma_0(f) \leq \operatorname{diam} Y,$$

$$(2) \qquad 0 \leq \sigma^*(f) \leq \sigma_0^*(f) \leq \sigma_0(f) \leq \operatorname{diam} Y,$$

$$(3) \qquad \sigma(f) = \operatorname{Sup}\{\sigma^*(f \mid A): A \subset X, A \neq \varnothing \text{ connected}\},$$

$$(4) \qquad \sigma_0(f) = \operatorname{Sup}\{\sigma_0^*(f \mid A): A \subset X, A \neq \varnothing \text{ connected}\}.$$

For $\tau = \sigma, \sigma^*, \sigma_0, \sigma_0^*$, the corresponding spans $\tau(X)$ of a space X are the spans $\tau(\mathrm{id}_X)$ of the identity mapping on X. The span $\sigma(f)$ of a