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WEAK*-CLOSED COMPLEMENTED INVARIANT SUBSPACES OF $L_\infty(G)$ AND AMENABLE LOCALLY COMPACT GROUPS

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One of the main results of this paper implies that a locally compact group G is amenable if and only if whenever X is a weak*-closed left translation invariant complemented subspace of $L_\infty(G)$, X is the range of a projection on $L_\infty(G)$ commuting with left translations. We also prove that if G is a locally compact group and M is an invariant W^*-subalgebra of the von Neumann algebra VN(G) generated by the left translation operators l_g, $g \in G$, on $L_2(G)$, and $\Sigma(M) = \{g \in G; l_g \in M\}$ is a normal subgroup of G, then M is the range of a projection on VN(G) commuting with the action of the Fourier algebra $A(G)$ on VN(G).

1. Introduction. Let G be a locally compact group and $L_\infty(G)$ be the algebra of essentially bounded measurable complex-valued functions on G with pointwise operations and essential sup norm. Let X be a weak*-closed left translation invariant subspace of $L_\infty(G)$. Then X is *invariantly complemented* in $L_\infty(G)$ if X admits a left translation invariant closed complement, or equivalently, X is the range of a continuous projection on $L_\infty(G)$ commuting with left translations.

H. Rosenthal proved in [**13**] that if G is an abelian locally compact group and X is a weak*-closed translation invariant complemented subspace of $L_\infty(G)$, then X is invariantly complemented in $L_\infty(G)$. Recently Lau [**11**, Theorem 3.3] proved that a locally compact group G is left amenable if and only if every left translation invariant weak*-closed subalgebra of $L_\infty(G)$ which is closed under conjugation is invariantly complemented. Note that if T is the circle group, then the Hardy space H_∞ is a weak*-closed translation invariant subalgebra of $L_\infty(T)$ and *not* (invariantly) complemented (see [**15**] and Corollary 4).

In [**20**, Lemma 4], Y. Takahashi proved that if G is a compact group, then any weak*-closed complemented left translation invariant subspace of $L_\infty(G)$ is invariantly complemented. However, there is a gap in Takahashi's adaptation of Rosenthal's argument (see Zentralblatt für Mathematik 1982: 483.43002). It should be observed that Rosenthal's original argument in [**13**, Theorem 1.1] is valid only for locally compact