PACIFIC JOURNAL OF MATHEMATICS
Vol 123, No. 1, 1986

ON A PROBLEM OF GAUSS-KUZMIN TYPE FOR CONTINUED FRACTION WITH ODD PARTIAL QUOTIENTS

SOFIA KALPAZIDOU

Let x be a number of the unit interval. Then x may be written in a unique way as a continued fraction

$$x = 1/(\alpha_1(x) + \varepsilon_1(x)/(\alpha_2(x) + \varepsilon_2(x)/(\alpha_3(x) + \cdots)))$$

where $\varepsilon_n \in \{-1, 1\}$, $\alpha_n \geq 1$, $\alpha_n \equiv 1 \pmod 2$ and $\alpha_n + \varepsilon_n > 1$. Using the ergodic behaviour of a certain homogeneous random system with complete connections we solve a variant of Gauss-Kuzmin problem for the above expansion.

1. Introduction. We define continued fraction with odd partial quotients as follows. Let us partition the unit interval into

$$\left(\frac{1}{2k}, \frac{1}{2k-1}\right], \quad k = 1, 2, \ldots, \quad \text{and} \quad \left(\frac{1}{2k-1}, \frac{1}{2k-2}\right], \quad k = 2, 3, \ldots$$

and define the transformation T: $[0,1] \to [0,1]$ by

$$Tx = e\left(\frac{1}{x} - (2k-1)\right)$$

where

$$e = 1 \quad \text{if } x \in \left(\frac{1}{2k}, \frac{1}{2k-1}\right],$$

and

$$e = -1 \quad \text{if } x \in \left(\frac{1}{2k-1}, \frac{1}{2k-2}\right].$$

We arrive at

$$x = \frac{1}{2k - 1 + e(Tx)}$$

and therefore the map T generates a continued fraction

$$(1.1) \qquad x = \frac{1}{\alpha_1(x) + \varepsilon_1(x)/(\alpha_2(x) + \varepsilon_2(x)/(\alpha_3(x) + \cdots))}$$

$$= \begin{bmatrix} 1, \ \varepsilon_1(x), \ \varepsilon_2(x), \ldots \\ \alpha_1(x), \ \alpha_2(x), \ \alpha_3(x), \ldots \end{bmatrix}$$