PACIFIC JOURNAL OF MATHEMATICS
Vol. 123, No. 2, 1986

AVERAGING PROPERTIES OF PLURIHARMONIC BOUNDARY VALUES

WADE C. RAMEY

Suppose $D \subset C^n$ is a smoothly bounded domain and u is bounded and pluriharmonic in D. Let u^* denote the boundary function of u, and let $\zeta_0 \in \partial D$. It is shown that if u^* has good averaging behavior on one curve in ∂D through ζ_0, then u^* has good averaging behavior on all curves in ∂D through ζ_0, provided the curves in question satisfy a certain directional condition. These results fail if the directional condition on the curve is violated.

I. Introduction. Let D be a domain in $\mathbf{C}^n$ with C^1-boundary, and for $\zeta \in \partial D$, let $T^C_{\partial D}(\zeta)$ denote the complex tangent space of ∂D at ζ. If f is a complex valued function defined in D, we denote by $f^*(\zeta)$ the nontangential limit of f at ζ, provided this limit exists.

Fix a point $\zeta_0 \in \partial D$. We will be interested in C^3-curves $\gamma\colon (-1,1) \to \partial D$ such that

$$\gamma(0) = \zeta_0, \quad \gamma'(0) \notin T^C_{\partial D}(\zeta_0). \tag{1}$$

Note that since $T^C_{\partial D}(\zeta_0)$ is of (real) codimension 1 within the full tangent space of ∂D at ζ_0, the "typical" smooth curve in ∂D through ζ_0 will satisfy the last condition in (1).

The results of this paper are concerned with averaging properties of pluriharmonic boundary values along such curves. The main thrust of these results is that if good averaging behavior occurs on one curve satisfying (1), then the same must be true of *every* curve satisfying (1). We first take up the case of H^∞-boundary values.

THEOREM 1. *If $f \in H^\infty(D)$, and if*

$$\lim_{h \to 0^+} \frac{1}{h} \int_0^h f^*(\gamma(x))\, dx = L \tag{2}$$

for one curve γ satisfying (1), *then* (2) *is true for every curve γ satisfying* (1).

Note that Theorem 1 also tells us that if (2) is true, then

$$\lim_{h \to 0^+} \frac{1}{h} \int_{-h}^0 f^*(\gamma(x))\, dx = L$$