PACIFIC JOURNAL OF MATHEMATICS
Vol. 123, No. 2, 1986

AN EXTENSION OF SINGULAR HOMOLOGY TO BANACH ALGEBRAS

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By making use of a simple connection with Banach algebras we introduce certain relations into singular homology and cohomology at the chain level and show that we obtain homology and cohomology theories. The deviation between singular and the new theory is measured by what turns out to be another homology theory HM. One of the main results is that HM is zero on simplicial complexes but not on metric spaces in general. This shows that for any coefficient group there are an infinite number of different homology theories agreeing with the associated homology theory on simplicial complexes.

Section 2 shows that HM detects all the anomalous singular homology constructed by Barratt and Milnor in [**BM**]. Section 3 gives a simple application to co-products and shows that we get the usual addition formula in homology for co-products without the assumption of a co-identity.

The main applications of this theory will be in a subsequent paper where the same relations are introduced into homotopy theory. The results of the present paper will show that the Hurewicz map factors through these new groups. Another application will be a nice (i.e. computable) way of relating the algebraic structure of $[X, H]$ (H an H-space) with properties of the maps induced by elements of $[X, H]$ in homology and cohomology.

1. In this section we will introduce a functor $A\colon \mathscr{T} \to \mathscr{C}$ where $\mathscr{T}$ is the category of spaces and continuous maps and $\mathscr{C}$ is the category of chain complexes of abelian groups and chain maps. This functor will be very similar to the functor S which assigns to each space X the singular complex SX (see [**G**]).

For any two complex algebras B_1 and B_2 we defiine $L(B_1, B_2)$ to be the group of all complex linear maps from B_1 and B_2 under pointwise addition. Recall that for a space X, the group $S_q X$ in the singular chain complex SX is the free abelian group on the set of all continuous maps from the q-simplex Δ_q into the space X. Every such map induces an algebra homomorphism from $C(X)$ to $C(\Delta_q)$, where $C(X)$ is the algebra