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PEAK POINTS IN BOUNDARIES NOT OF FINITE TYPE

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It is known that, in domains in $\mathbf{C}^2$ which are pseudoconvex and of finite type, compact subsets of peak sets for $A^\infty(D)$ are peak sets for $A^\infty(D)$. We give an example of a convex domain D (not of finite type) whose weakly pseudoconvex boundary points form a line segment K, with the property: K is a peak set for $A^\infty(D)$, but a point $p \in K$ is not a peak point for any $A^\alpha(D)$, $\alpha > 0$. We also consider briefly the case when the weakly pseudoconvex boundary points form a disc.

0. Introduction. Let D be a bounded pseudoconvex domain in $\mathbf{C}^n$ with C^∞ boundary, and let $A^\alpha(D)$ denote the algebra of functions holomorphic in D and of class C^α in $\overline{D}$; here $0 \leq \alpha \leq \infty$. A compact set $K \subset \partial D$ is a peak set for such an algebra A if there exists $f \in A$ so that $f = 1$ on K while $|f| < 1$ on $\overline{D} \setminus K$; f is said to be a peak function for K. (If a peak set is a singleton $\{p\}$, p is called a peak point.) If D is strongly pseudoconvex, Chaumat and Chollet have proved in [**3**] that every compact subset of a peak set for $A^\infty(D)$ is a peak set for $A^\infty(D)$. In [**5**] it was shown that this also holds for domains in $\mathbf{C}^2$ of finite type. (Recall that $D \Subset \mathbf{C}^2$ is of finite type if one-dimensional complex manifolds cannot be tangent to ∂D to arbitrarily high order.) If ∂D is allowed to contain a complex manifold, it is easy to see that compact subsets of peak sets need not be peak sets (cf. Example 2.2 below). The main purpose of this paper is to show that compact subsets can fail to be peak sets even if $\partial D \Subset \mathbf{C}^2$ contains no complex manifold.

A closely related question is whether points of ∂D are peak points for some $A^\alpha(D)$. It is known ([**4**], [**6**]) that, if p is a point of strong pseudoconvexity, then p is a peak point for $A^\infty(D)$. In the case of points of weak pseudoconvexity, the following fact is an immediate consequence of a paper of Bedford and Fornæss [**1**]: If $D \Subset \mathbf{C}^2$ is of finite type, each point p is a peak point for some $A^\alpha(D)$, where $\alpha = \alpha(p)$ is positive, but it may approach zero as the geometry of ∂D allows complex manifolds tangent to higher and higher order at p. The main example below shows that this degeneracy of α is reasonable since, if complex manifolds can be tangent to arbitrarily high order at p, p may fail to be a peak point for $\bigcup_{\alpha > 0} A^\alpha(D)$.